(1)

פתרון מבחן המתמטיקה לכלכלנים ה'
תשע"ב סמ' א' מועד ב'

שאלה 1:

א.

$$\int \frac{e^{\frac{1}{x}}}{x^2}dx \underset{\substack{t=\frac{1}{x}\\ dt=-\frac{1}{x^2}dx}}{=} -\int e^t\,dt = -e^t + C = \boxed{-e^{\frac{1}{x}}+C}$$

ב.

$$\int (4-3x)\cdot e^{-3x}\,dx = (4-3x)\frac{e^{-3x}}{-3} - \int e^{-3x}dx$$

$f = 4-3x$, $g' = e^{-3x}$

$f' = -3$, $g = \frac{e^{-3x}}{-3}$

$$= \frac{4-3x}{-3}e^{-3x} - \frac{e^{-3x}}{-3} + C$$

$$= (x-1)e^{-3x} + C$$

ג.

$$\int \frac{2x+3}{x(x^2-2x-3)}dx = \int \frac{2x+3}{x(x-3)(x+1)}dx \overset{3.\text{ח}}{=} (*)$$

$$\frac{2x+3}{x(x-3)(x+1)} = \frac{a}{x} + \frac{b}{x-3} + \frac{c}{x+1} \overset{3.\text{ח}}{=} \frac{a(x-3)(x+1) + bx(x+1) + cx(x-3)}{x(x-3)(x+1)}$$

$x=0$: $3 = -3a \Rightarrow a = -1$

$x=3$: $9 = 12b \Rightarrow b = \frac{3}{4}$

$x=-1$: $1 = 4c \Rightarrow c = \frac{1}{4}$

$$\Rightarrow = -\frac{1}{x} + \frac{\left(\frac{3}{4}\right)}{x-3} + \frac{\left(\frac{1}{4}\right)}{x+1}$$

$$(*) = \int -\frac{1}{x} + \frac{\left(\frac{3}{4}\right)}{x-3} + \frac{\left(\frac{1}{4}\right)}{x+1}dx = -\ln|x| + \frac{3}{4}\ln|x-3| + \frac{1}{4}\ln|x+1| + C$$

$$f(x,y)=(x^2+y^2)e^{-y}$$

2 [illegible]

(1) $f'_x=2x\cdot e^{-y}=0$

(2) $f'_y=2y\cdot e^{-y}+(x^2+y^2)\cdot e^{-y}(-1)=e^{-y}(2y-x^2-y^2)=0$

$(1)\Rightarrow x=0 \xRightarrow{(2)} 2y-y^2=0 \Rightarrow y(2-y)=0 \Rightarrow y=0 \;\&\; y=2$

נק': $(0,2)$, $(0,0)$

$A=f''_{xx}=2e^{-y}$

$B=f''_{xy}=2x\cdot e^{-y}(-1)$

$C=f''_{yy}=-e^{-y}(2y-x^2-y^2)+e^{-y}(2-2y)=e^{-y}(-2y+x^2+y^2+2-2y)$

	$(0,0)$	$(0,2)$
$A=2e^{-y}$	2	$\frac{2}{e^2}\approx 0.27$
$B=-2xe^{-y}$	0	0
$C=e^{-y}(x^2+y^2-4y+2)$	2	$\frac{-2}{e^2}$
$D=AC-B^2$	$D=4>0$ $A>0$ min	$D=\frac{-4}{e^4}<0$ אוכף

~~max~~ אוכף $-(0,2,\frac{4}{e^2})$, $\min-(0,0,0)$:תשובה

שאלה 3: $f(x,y)=x^2-2x+y^2-4y$ בתחום: $x^2+y^2\leq 4$

נק' חשודות בפנים:

$f'_x=2x-2=0 \Rightarrow x=1$

$f'_y=2y-4=0 \Rightarrow y=2 \Rightarrow (1,2) \leftarrow$ לא בתחום

נק' חשודות על השפה: $L(x,y,\lambda)=x^2-2x+y^2-4y-\lambda(x^2+y^2-4)$

(1) $L'_x=2x-2-2\lambda x=0 \Rightarrow \frac{2x-2}{2x}=\lambda$

(2) $L'_y=2y-4-2\lambda y=0 \Rightarrow \frac{2y-4}{2y}=\lambda$

(3) $x^2+y^2=4$

$$\Rightarrow \frac{x-1}{x}=\frac{y-2}{y} \Downarrow (x-1)y=x(y-2)$$

$$xy-y=xy-2x \Downarrow y=2x \xRightarrow{(3)}$$

$x^2+4x^2=4$

$5x^2=4$

$x^2=\frac{4}{5} \Rightarrow x=\pm\frac{2}{\sqrt5} \Rightarrow \boxed{\left(\frac{2}{\sqrt5},\frac{4}{\sqrt5}\right),\left(-\frac{2}{\sqrt5},-\frac{4}{\sqrt5}\right)}$

$f\left(\frac{2}{\sqrt5},\frac{4}{\sqrt5}\right)=\frac{4}{5}-\frac{4}{\sqrt5}+\frac{16}{5}-\frac{16}{\sqrt5}=\frac{20}{5}-\frac{20}{\sqrt5}=4-\frac{20}{\sqrt5}$ מינ' min ≈ -4.94

$f\left(-\frac{2}{\sqrt5},-\frac{4}{\sqrt5}\right)=\frac{4}{5}+\frac{4}{\sqrt5}+\frac{16}{5}+\frac{16}{\sqrt5}=4+\frac{20}{\sqrt5}$ max ≈ 12.94

(4)

<u>שאלה 4</u> א.

$\nearrow +$ $\quad$ $- \searrow$

$(-2,2)$

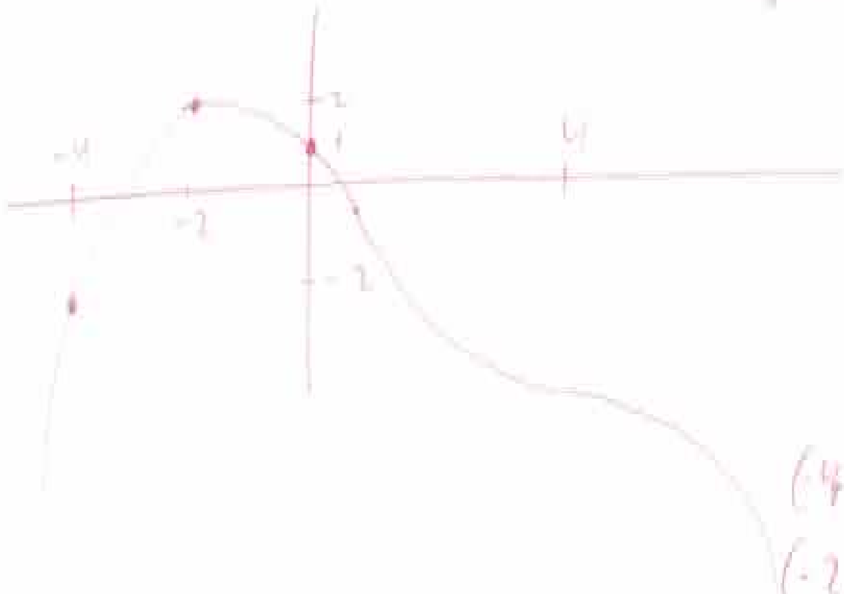
MAX

ב.

$\cap \quad | \quad \cup \quad | \quad \cap$

פיתול 1 $\quad$ 4

<u>סיכום</u>



ג. 2 נק' קיצון

ד. $x=-4$ מינימום $(-4,-4)$

$x=-2$ מקסימום $(-2,2)$

ה.

	+	+	-	-
f''	-	+	+	-
	1	3	4	

$x \leq 1$ או $3 \leq x \leq 4$.

(5)

שאלה 15: מספר הנוסעים x. $24 \le x \le 50$

א. $90-3\cdot(x-24)=90-3x+72=162-3x$

ב. $f(x)=x(162-3x)-\left(\frac{x^3}{24}-3x^2+800\right)$

$=162x-3x^2-\frac{x^3}{24}+3x^2-800$

$=-\frac{x^3}{24}+162x-800$

$f'(x)=-\frac{3x^2}{24}+162=-\frac{x^2}{8}+162=0$

$x^2=162\cdot 8=81\cdot 2\cdot 8=9^2\cdot 4^2$

$x=\pm 36 \Rightarrow x=36$

$f(24)=2512$

$f(36)=3088$ max

$f(50)=2091.67$

⑥

$$y' - 2y = 3x \;/ \cdot e^{\int -2\,dx} = e^{-2x}$$. 6 :6 ✓ 16

$$(y e^{-2x})' = 3x e^{-2x} \;/ \int$$

$$y e^{-2x} = 3\int \underset{f}{x}\, \underset{g'}{e^{-2x}}\,dx = 3\cdot\left[\frac{x}{-2}e^{-2x} + \frac{1}{2}\int e^{-2x}\,dx\right]$$

$$f' = 1 \quad g = \frac{e^{-2x}}{-2}$$

$$= -\frac{3}{2}xe^{-2x} + 3\cdot\frac{1}{2}\cdot\frac{e^{-2x}}{-2} + C = -\frac{3}{2}xe^{-2x} - 3\frac{e^{-2x}}{4} + C$$

$$\boxed{y = -\frac{3}{2}x - \frac{3}{4} + C\cdot e^{2x}}$$

$$y(0) = -\frac{3}{4} + C = 1 \Rightarrow C = 1\frac{3}{4}$$ .2

$$\Rightarrow \boxed{y = -\frac{3}{2}x - \frac{3}{4} + \frac{7}{4}e^{2x}}$$

$$x = 2 \Leftarrow x = \pm 2 \Leftarrow \underset{x^2 = 4}{\cancel{}} \Leftarrow \frac{x}{4} = \frac{1}{x}$$:7 ✓ 16

$$x = 1 \Leftarrow x = \pm 1 \Leftarrow x = \frac{1}{x}$$

$$\int_0^1 x - \frac{x}{4}\,dx + \int_1^2 \frac{1}{x} - \frac{x}{4}\,dx = \frac{x^2}{2} - \frac{x^2}{8}\Big|_0^1 + \ln|x| - \frac{x^2}{8}\Big|_1^2$$

$$= \frac{1}{2} - \frac{1}{8} + \left(\ln(2) - \frac{1}{2}\right) - \left(0 - \frac{1}{8}\right)$$

7

$$= -\frac{1}{8} + \ln 2 + \frac{1}{8} = \ln 2 \approx 0.693$$