מתמטיקה לכלכלנים ב' - 8/7/10

<u>תש"ע</u> <u>סמס' ב'</u> <u>מועד ב'</u>

<u>חלק א' שאלות חובה</u>

1. א.

$$\int \frac{(5\ln x+3)^2}{x}dx \underset{t=\ln x}{=} \int (5t+3)^2 dt$$

$$= \frac{(5t+3)^3}{5\cdot 3} + C = \boxed{\frac{(5\ln x+3)^3}{15} + C}$$

ב.

$$\int \frac{x^3-3x^2+4x-3}{x^2-3x+2}dx = \int x + \frac{2x-3}{x^2-3x+2}dx$$

$$\begin{array}{l} \quad x \\ \hline x^3-3x^2+4x-3 \;\big)\; x^2-3x+2 \\ x^3-3x^2+2x \\ \hline \quad 2x-3 \end{array}$$

$$= \boxed{\frac{x^2}{2} + \ln|x^2-3x+2| + C}$$

$$x_{1,2} = \frac{3\pm\sqrt{9-8}}{2} = 2,1$$ ט"ג

$$\frac{2x-3}{(x-2)(x-1)} = \frac{a}{x-2} + \frac{b}{x-1} = \frac{a(x-1)+b(x-2)}{(x-2)(x-1)}$$

$x=1$; $-1 = -b \Leftarrow b=1$

$x=2$; $1 = a$

$$\int \frac{1}{x-2} + \frac{1}{x-1}dx = \ln|x-2| + \ln|x-1|$$

$$\int \frac{\ln x}{\sqrt{x}} = \int \underset{g'}{\underset{\shortparallel}{x^{-\frac{1}{2}}}} \cdot \underset{f}{\underset{\shortparallel}{\ln x}}\, dx = 2x^{\frac{1}{2}} \ln x - 2\int x^{-\frac{1}{2}} dx =$$

$$g = 2x^{\frac{1}{2}} \quad f' = \frac{1}{x}$$

$$= \boxed{2x^{\frac{1}{2}} \ln x - 2 \cdot 2 \cdot x^{\frac{1}{2}} + C}$$

.2

$$f(x,y) = 3x^3 - xy^2 + y^2 + 7x$$

$$f'_x = 9x^2 - y^2 + 7 = 0$$

$$f'_y = -2xy + 2y = 0 \Rightarrow -2y(x-1) = 0$$

$y=0$ או $x=1$

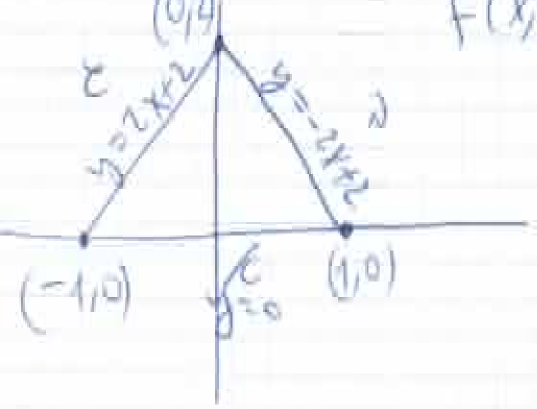
$y=0 \Rightarrow 9x^2+7=0$ אין פתרון.

$x=1 \Rightarrow 9-y^2+7=0 \Rightarrow y^2=16$

$y = \pm 4$

$(1,4)$, $(1,-4)$

	$(1,4)$	$(1,-4)$
$A = f''_{xx} = 18x$	18	18
$B = f''_{xy} = -2y$	-8	8
$C = f''_{yy} = -2x+2$	0	0
	$D=-64<0$ אוכף	$D=-64<0$ אוכף



$f(x,y) = xy - x$, 3

$m = \frac{2}{-1} = -2$, ב

$y = -2\cdot(x-1) = -2x+2$

$m = \frac{2}{1} = 2$, ג

$y = 2(x+1) = 2x+2$

$f'_x = y - 1 = 0 \Rightarrow y = 1$, $f'_y = x = 0 \Rightarrow \boxed{(0,1)}$

פנימ'

$f(x,0) = -x \Rightarrow f' = -1 \neq 0$ — א קט

$f(x,-2x+2) = x(-2x+2) - x = -2x^2 + x$ — ב קט

$f' = -4x + 1 = 0 \Rightarrow 1 = 4x \Rightarrow x = \frac{1}{4}$, $y = -2\cdot\frac{1}{4} + 2 = \frac{3}{2}$

$\checkmark \boxed{\left(\frac{1}{4}, \frac{3}{2}\right)}$

$f(x,2x+2) = x(2x+2) - x = 2x^2 + x$ — ג קט

$f' = 4x + 1 = 0 \Rightarrow x = -\frac{1}{4}$, $y = 2\left(-\frac{1}{4}\right) + 2 = \frac{3}{2}$

$\checkmark \boxed{\left(-\frac{1}{4}, \frac{3}{2}\right)}$

$f(0,1) = 0$

$f\left(\frac{1}{4}, \frac{3}{2}\right) = \frac{1}{4}\cdot\frac{3}{2} - \frac{1}{4} = \frac{3}{8} - \frac{2}{8} = \frac{1}{8}$

$f\left(-\frac{1}{4}, \frac{3}{2}\right) = -\frac{1}{4}\cdot\frac{3}{2} + \frac{1}{4} = -\frac{3}{8} + \frac{2}{8} = -\frac{1}{8}$

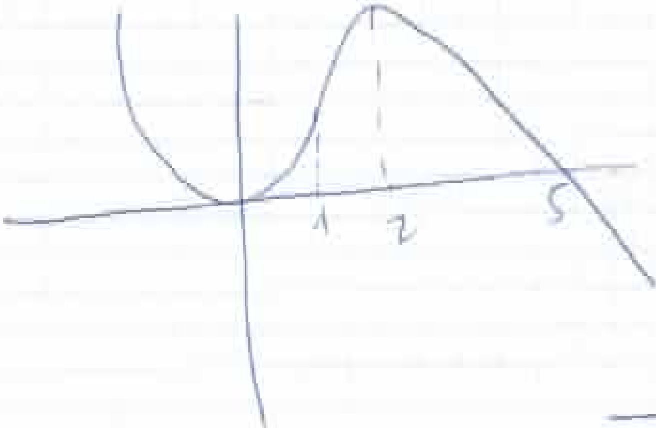
$f(-1,0) = 1$ max

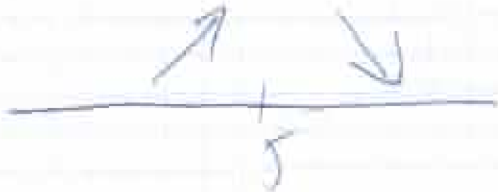
$f(1,0) = -1$ min

$f(0,2) = 0$

4

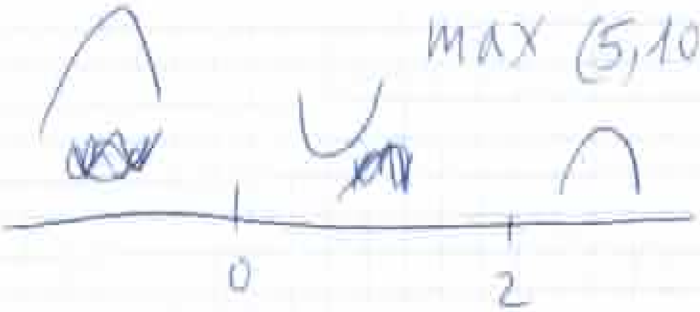
א.





max (5,10)

ב.



ג. $\lim_{x\to-\infty} f(x) = -\infty$

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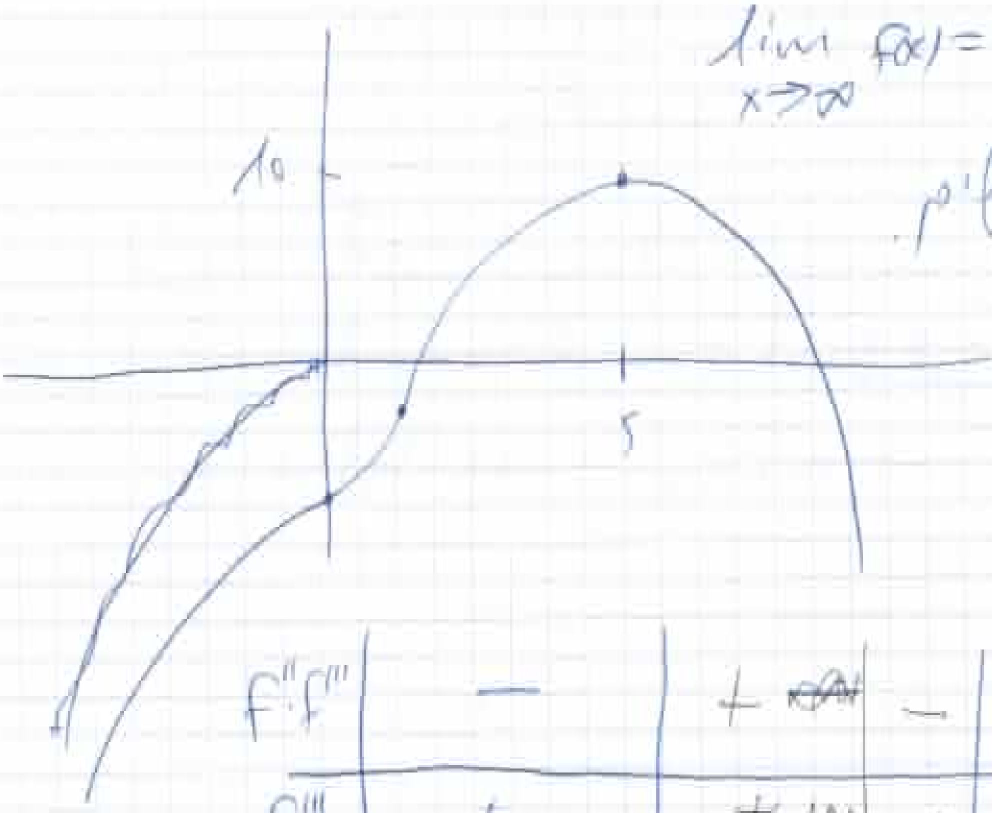
ד. על לולים



ה.

f'' f'''	−	+	−	+
f'''	+	+	−	−
f''	−	+	+	−
		0	1	2

ה.

$x \le 0$ או $1 \le x \le 2$

אלקה

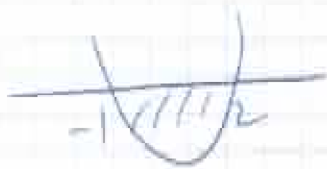
5. $y = \ln(-x^2+x+2)$

א. ת"ה

$-x^2+x+2>0$

$x^2-x-2<0$

$x_{1,2} = \frac{1 \pm \sqrt{1+8}^{\,3}}{2} = 2, -1$



$-1<x<2$

ב. נק' קיצון מוחלטות בקטע $[0,1]$.

$y' = \frac{-2x+1}{-x^2+x+2} = 0 \Rightarrow x = \frac{1}{2}$

$f(0) = \ln(2)$ min

$f(\frac{1}{2}) = \ln(-\frac{1}{4}+\frac{1}{2}+2) = \ln(2\frac{1}{4})$ max

$f(1) = \ln(-1+1+2) = \ln 2$ min

$y' - y = x \quad / \cdot e^{\int -1\,dx} = e^{-x}$.א .6

$(y \cdot e^{-x})' = x \cdot e^{-x} \quad / \int$

$y \cdot e^{-x} = \int \underset{f}{x} \cdot \underset{g'}{e^{-x}}\,dx = -xe^{-x} + \int e^{-x}\,dx$

$f' = 1 \quad g = -e^{-x}$

$\boxed{y \cdot e^{-x} = -xe^{-x} - e^{-x} + C}$

$y=0$, $x=0$.ב

$0 \cdot 1 = -0 \cdot 1 - 1 + C$

$0 = -1 + C \Rightarrow C = 1$

$\boxed{y \cdot e^{-x} = -x \cdot e^{-x} - e^{-x} + 1}$

$x=1 \Leftarrow 2x=2 \Leftarrow x=2-x \Leftarrow \sqrt{x}=\sqrt{2-x}$ .7

$S = \int_0^1 \sqrt{x}\,dx + \int_1^2 \sqrt{2-x}\,dx = \frac{2}{3}x^{\frac{3}{2}}\Big|_0^1 - \frac{2}{3}(2-x)^{\frac{3}{2}}\Big|_1^2$

$= \frac{2}{3} - \frac{2}{3} \cdot [0-1] = \frac{4}{3}$