פתרון מבחן במתמטיקה לטכניון ב'
תשע"ג סמ' ב' מועד ב'

שאלה 1:

א. $$\int \frac{dx}{1+\sqrt{x}} \underset{\substack{t=\sqrt{x} \\ t^2=x \\ 2t\,dt=dx}}{=} \int \frac{2t\,dt}{1+t}$$

$$= \int \frac{2t+2-2}{t+1}dt = \int 2 - \frac{2}{t+1}dt$$

$$= 2t - 2\ln|t+1| + C = 2\sqrt{x} - 2\ln(\sqrt{x}+1) + C$$

ב. $$\int \frac{\ln x}{x^2}dx = \int \underset{g'}{\underset{\shortparallel}{x^{-2}}} \underset{f}{\underset{\shortparallel}{\ln x}}\,dx = -\frac{1}{x}\cdot\ln x + \int \frac{1}{x^2}dx$$

$$g = \frac{x^{-1}}{-1} \quad f' = \frac{1}{x}$$

$$= \boxed{-\frac{\ln x}{x} - \frac{1}{x} + C}$$

ג. $$\int \frac{x^2+2}{x(x^2-6x+9)}dx = \int \frac{x^2+2}{x(x-3)^2}dx = \int \frac{\frac{2}{9}}{x} + \frac{\frac{7}{9}}{x-3} + \frac{\frac{11}{3}}{(x-3)^2}dx$$

$$\frac{x^2+2}{x(x-3)^2} = \frac{a}{x} + \frac{b}{x-3} + \frac{c}{(x-3)^2} = \frac{a(x-3)^2 + bx(x-3) + cx}{x(x-3)^2}$$

$a = \frac{2}{9} \Leftarrow 2 = 9a \Leftarrow x$

$b = \frac{7}{9} \Leftarrow 1 = a+b$: מקדם x^2 $\quad c = \frac{11}{3} \Leftarrow 11 = 3c \Leftarrow x$

$\frac{2}{9}\ln|x| + \frac{7}{9}\ln|x-3| - \frac{11}{3}\cdot\frac{1}{x-3} + C$ סוף:

שאלה 2:

$f(x,y) = xy - x^2 - y^2 - 2x - 2y + 4$

$f'_x = y - 2x - 2 = 0 \,/\cdot 2 \qquad 2y - 4x - 4 = 0$ +

$f'_y = x - 2y - 2 = 0 \qquad -2y + x - 2 = 0$

$-3x = 6$

$x = -2$

$y + 4 - 2 = 0 \Rightarrow y = -2 \Leftarrow$

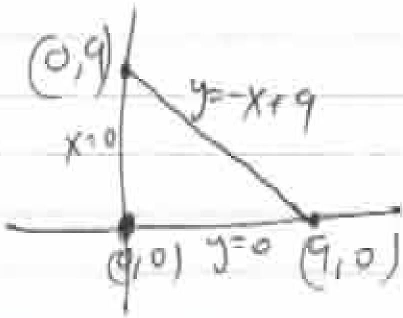
$A = f''_{xx} = -2$

$B = f''_{xy} = 1 \Rightarrow D = 3 > 0,\ A = -2 < 0$

$C = f''_{yy} = -2 \qquad \boxed{\max(-2,-2)} \Leftarrow$

$\max(-2,-2,8)$

$$f(x,y)=2+2x+2y-x^2-y^2$$ שאלה 3:



נק' פנימית:

$$f'_x=2-2x=0 \Rightarrow x=1$$
$$f'_y=2-2y=0 \Rightarrow y=1$$
$$\boxed{(1,1)}$$

נק' על $x=0$:

$$f(0,y)=2+2y-y^2 \Rightarrow f'=2-2y=0 \Rightarrow y=1$$
$$(0,1)$$

על $y=0$:

$$f(x,0)=2+2x-x^2 \Rightarrow f'=2-2x=0 \Rightarrow x=1$$
$$\boxed{(1,0)}$$

על $y=9-x$:

$$f(x,9-x)=2+2x+2(9-x)-x^2-(9-x)^2$$
$$=2+2x+18-2x-x^2-(81-18x+x^2)$$
$$=-61-x^2+18x-x^2=-2x^2+18x-61$$
$$f'=-4x+18 \Rightarrow x=\frac{18}{4}=\frac{9}{2}, \quad y=\frac{9}{2} \quad \boxed{\left(\frac{9}{2},\frac{9}{2}\right)}$$

$f(1,1)=2+2+2-1-1=4$ max

$f(0,1)=2+2-1=3$

$f(1,0)=2+2-1=3$

$f\left(\frac{9}{2},\frac{9}{2}\right)=2+9+9-\frac{81}{4}-\frac{81}{4}=20-\frac{81}{2}=20-40\frac{1}{2}=-20\frac{1}{2}$

$f(0,0)=2$, $f(0,9)=2+18-81=-61$ min, $f(9,0)=2+18-81=-61$ min

שאלה 4:

א.

ב.

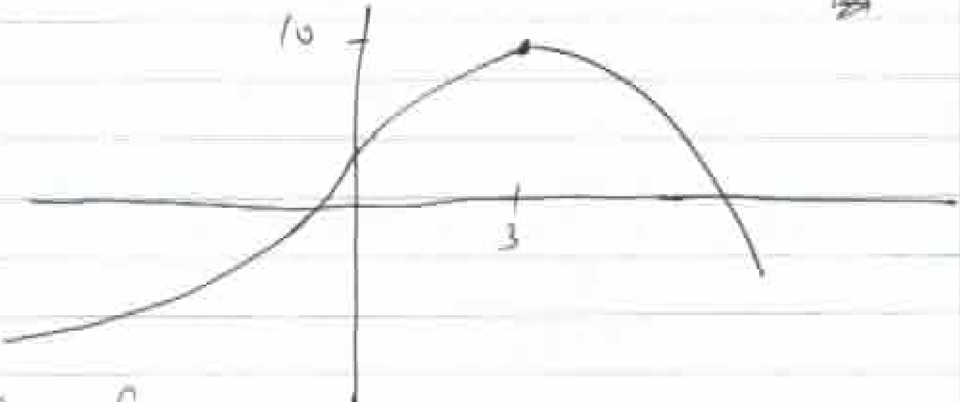
ג.

$\lim_{x\to\infty} f(x) = -\infty$, ב-∞- לא ניתן לקבוע

ב:

עולה קמור.

ד.



ד. ב $[-1,2]$ הפונקציה עולה ולכן
ב-1- מינימום וב-2 מקסימום.

ה.

f''	+	−	−
f'	+	+	−
	-1	3	

$\boxed{-1 \le x \le 3}$

שאלה 5:

$0 \le x \le 20$,

$$f(x) = (2x+3)^{\frac{3}{2}} - \frac{x^2}{2} - 4x - 4$$

רווח

$$f' = \frac{3}{2}(2x+3)^{\frac{1}{2}} \cdot 2 - x - 4 = 0$$

$$3\sqrt{2x+3} = x+4 \quad /\!/()^2$$

$$9(2x+3) = x^2+8x+16$$

$$0 = x^2 - 10x - 11$$

$$x_{1,2} = \frac{10 \pm \sqrt{100+44}^{\,12}}{2} = 11, -1$$

$$f(0) = 3^{\frac{3}{2}} - 4 = \sqrt{27} - 4 \approx 1.2$$

$$f(11) = 25^{\frac{3}{2}} - \frac{121}{2} - 44 - 4 = 125 - 60\tfrac{1}{2} - 48 = 16.5 \text{ ma}$$

$$f(20) = 43^{\frac{3}{2}} - 200 - 80 - 4 = \underset{281.95}{\sqrt{79{,}507}} - 284 \approx -2$$

min

$$y''+2y'+y=0$$ 6

$$k^2+2k+1=0$$

$$(k+1)^2=0 \Rightarrow k=-1$$

$$y=c_1e^{-x}+c_2x\cdot e^{-x}$$

$$y(0)=c_1=2$$

$$y(1)=2\cdot e^{-1}+c_2\cdot e^{-1}=\frac{3}{e} \;/\cdot e$$

$$2+c_2=3 \Rightarrow c_2=1$$

$$\boxed{y=2\cdot e^{-x}+x\cdot e^{-x}}$$

$$\sqrt{x}=2\sqrt{5-x}$$ 7

$$x=4(5-x)$$

$$x=20-4x \Rightarrow 5x=20$$

$$\boxed{x=4}$$

$$\int_0^4\sqrt{x}\,dx+\int_4^5 2\cdot\sqrt{5-x}\,dx=\frac{2}{3}x^{\frac{3}{2}}\Big|_0^4-\frac{2\cdot 2(5-x)^{\frac{3}{2}}}{3}\Big|_4^5$$

$$=\frac{16}{3}-\left[0-\frac{4}{3}\right]=\frac{20}{3}=6\frac{2}{3}$$