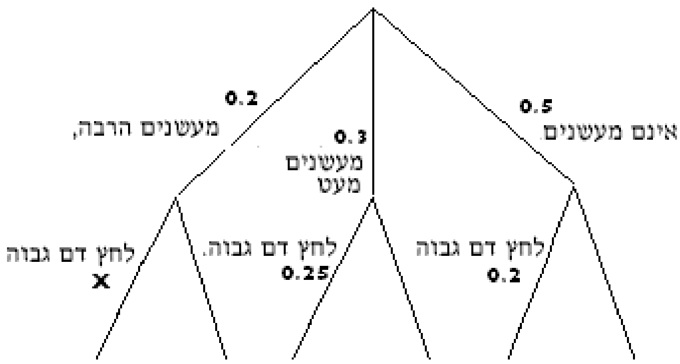
Solutions of the Final Exam B

Management of Technology

10.August.2005

Problem 1



a) Denote x - probability to have a high blood pressure in the group of heavy smokers.
Then $0.2x + 0.3 * 0.25 + 0.5 * 0.2 = 0.35$ and $x = 0.875$
b) Using formula of Bayes

P(Non-smokers/Healthy)=$\dfrac{\text{P(Healthy/Non-smokers)*P(Non-smokers)}}{\text{P(Healthy)}} =$

$\frac{0.8*0.5}{0.65} = 0.62$

c) $P(A) * P(B) = 0.5 * (1 - 0.35) = 0.325$

$P(A \cap B) = 0.5 * (1 - 0.2) = 0.4$

$P(A) * P(B) \neq P(A \cap B)$ ==> A and B - dependent events

Problem 2

a)

X\Y	0	2	3	P(X)
10	0.12	0.08	0.1	0.3
20	0.05	0.05	0.1	0.2
30	0.03	0.17	0.3	0.5
P(Y)	0.2	0.3	0.5	1

b) E[X]=22; Var[X]=76; E[Y]=2.1; Var[Y]=1.29

c) $P(X=10, Y=0) \neq P(X=10) * P(Y=0)$ ==> X and Y - dependent

E[X,Y]=49.8 $\neq$ E[X]*E[Y]=22*2.1 ==> X and Y - correlated

d) E[Z]=0.5*E[X]-E[Y]=0.5*22-2.1=8.9

e) $E[X/Y=0] = \sum X * P(X/Y=0) = \sum X * \dfrac{P(X \cap Y=0)}{P(Y=0)} =$

$\dfrac{1}{P(Y=0)} \sum X * P(X \cap Y) = \dfrac{1}{0.2}(10 * 0.12 + 20 * 0.05 + 30 * 0.03) = 15.5$

Problem 3

a) $\int\limits_{-\infty}^{\infty} f_X(x)dx = 1$, hence $\int\limits_{10}^{\infty} \dfrac{k}{x^2}dx = -\dfrac{k}{x}|_{10}^{\infty} = \dfrac{k}{10} = 1$, and $k = 10$

b) $F(X) = \int\limits_{10}^{x} \dfrac{10}{t^2}dt = 1 - \dfrac{10}{x} \quad (x > 10)$

c) $P(X > 20) = 1 - P(X < 20) = 1 - F(20) = 1 - (1 - \dfrac{10}{20}) = 0.5$

d) $p = P(X > 15) = 1 - (1 - \dfrac{10}{15}) = 2/3$, hence $X \sim B(6, 2/3)$

If 3 exactly, $P(X=3) = C_6^3 \left(\frac{2}{3}\right)^3 \left(\frac{1}{3}\right)^3 = 0.219\,48$

If at least 3,

$P(X \geq 3) = 1 - P(X \leq 2) = 1 - (P(X=0) + P(X=1) + P(X=2)) =$

$1 - \left[C_6^0 \left(\frac{2}{3}\right)^0 \left(\frac{1}{3}\right)^6 + C_6^1 \left(\frac{2}{3}\right)^1 \left(\frac{1}{3}\right)^5 + C_6^2 \left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^4\right] = 0.9$

Problem 4

$n = 100;\ p = 0.8$
$P(x < x_0) = 0.95;$
$\Phi\left(\dfrac{x_0 - np + 1/2}{\sqrt{np(1-p)}}\right) = \Phi\left(\dfrac{x_0 - 100 * 0.8 + 1/2}{\sqrt{100 * 0.8 * 0.2}}\right) = \Phi\left(\dfrac{x_0 - 79.5}{4.0}\right) = 0.95$
From the table $\Phi(1.645) = 0.95$, hence $\dfrac{x_0 - 79.5}{4.0} = 1.645$ and $x_0 = 4.0 * 1.645 + 79.5 \approx 87$

Problem 5

$\mu = 560;\ n = 6$
$\overline{X} = \frac{1}{n}\sum\limits_{i=1}^{n} X_i = \frac{1}{6}(558 + 557 + 556 + 561 + 562 + 560) = 559.0$
$\widehat{S} = \sqrt{\frac{1}{n-1}\sum\limits_{i=1}^{n}(X_i - \overline{X})^2} = \sqrt{\frac{1}{5}(1 + 4 + 9 + 4 + 9 + 1)} = 2.37$
Test mean value with $\alpha = 0.05;\ c = 560$

$$\begin{array}{l} H_0 : \mu = c \\ H_A : \mu \neq c \end{array};\ \ t_\alpha = t_{0.025}(5) = 2.57$$

$T = \dfrac{\overline{X} - c}{\widehat{S}/\sqrt{n}} = \dfrac{|559 - 560|}{2.37/\sqrt{6}} = 1.0335 < t_\alpha = 2.57$
$T < t_\alpha$; We can not reject H_0. No significant difference for mean value.
A manufacturing is good

Problem 6

$n = 100;\ c = 0.51;\ \widehat{p} = 0.49;\ \alpha = 0.04$

$$\begin{array}{l} H_0 : p = c \\ H_A : p < c \end{array};\ \ z_\alpha = z_{1-0.04} = -1.75$$

$Z = \dfrac{\widehat{p} - c}{\sqrt{\frac{c(1-c)}{n}}} = \dfrac{0.49 - 0.51}{\sqrt{\frac{0.51 * 0.49}{100}}} = -0.40$
$Z > z_{1-\alpha}$ ==> We can not reject H_0. A proportion in this city not less then in the whole country.