Why using factor analysis? (dedicated to the centenary of factor analysis)

Alexander S. Kaplunovsky

Research Center for Quantum Communication Engineering
Holon Academic Institute of Technology, 52 Golomb Str., Holon 58102, Israel
e-mail: alex_ka@netvision.net.il

Abstract

100 years ago Charles Spearman published the paper about the factor of a general intelligence, being situated on the study of the correlation matrix of tests connected with intellectual functions. In our days, factor analysis has become an important statistical instrument of investigation in modern science, being an adequate tool to investigate the principles of interaction of components and their integration into a system. This approach to the study of the form of organization, called integratism, proposes the dismembering the system to correlated elements, analyzing their cross-relations and picking out system-forming elements, their relations and hierarchy. Therefore, the requested mathematical technique should obey all these requirements. Properties of factor analysis as a method of integratism, are considered and discussed. The history of factor analysis and its various modifications are reviewed on the sample of 3460 publications.

1 Introduction

Celebrating the 100-th anniversary of factor analysis, one can ask why this approach has become in our days a principal statistical method of investigation in life sciences. What are the reasons of its wide dissemination in almost all scientific fields? Why so much efforts are directed towards new modifications and development of factor analysis methods? These questions can be answered from the positions of contemporary scientific methodology.

While for most exact sciences, the using of differential equations, algebra, set theory, mathematical logic, and operational research is typical and usually sufficient, in such sciences as biology, psychology, sociology - methods intended to the analysis of multiple processes distribution and based on the probabilistic, rather then functional, homomorphism of the model and the object, are used. Therefore, the primary role passes on to the methods of mathematical statistics, theory of information, theory of random processes, etc. However, most of these methods has one crucial defect: they are not integral, giving answer to the question about the specifics and the reasons of organization of analyzed elements to a one system.

The only acceptable method realizing the principles of system approach and meeting all above-mentioned requests, is factor analysis as an integral statistical method, with its opportunity to define and evaluate the structural-functional organization of the system.

2 Methodology of integratism

The history of science knows two fundamental approaches to complex systems: reductionism and organicism [1]. Reductionism is based on the idea that investigation of the object must be performed by separating the object into its components and analysis their nature and properties. Organicism allows one to consider as an object of the study only the whole system without dividing it into elements with functions not reflecting directly to investigated properties of the system. The terms "holistic" and "phenomenological" are closely related with the concept "organic" [2].

At certain stages of the knowledge, these approaches were adequate and sufficient. In the process of historical development, reductionism as a method of cognition has found its realization in the methods of morphological or functional description of elements without consideration a role, hierarchy or relations of the elements in the system (systemic approach in its narrow sense). The fundamental advantages of this approach are the definition of a type of system giving an opportunity to choose an adequate method for analysis, and the definition of system-forming classes, boundaries of their existence and hierarchy. Nevertheless, a multitude of elements without functional relations between them, represents only a regulated or unregulated set of elements. Contemporary sciences deal with systems characterized by relations of developing and control and acting under laws of probability. Therefore, the revealing relations, their limits and hierarchy should be a principal goal in analysis of complicated systems. In spite of huge difficulties of analysis and data processing, only studying the whole complex of all specific causal-consequence relations, even seeming to be random and unimportant, can lead to correct conclusions.

It is important not to forget that the structure of system can be shown only on a set of its elements. It means that the choice of system attributes must be done according to principles of a systemic-structural approach or systemic approach in its wide sense [3]. Otherwise, the model will be in the best case isomorphic to a structure of the system, but having no information about elements of the system and their properties.

Thus we come to such principle of analysis, which allows us to choice system-forming elements on a basis of knowledge of all necessary system-forming relations, combines all advantages of reductionism and organicism, and makes a step from the most primitive, elementary levels backward to the levels with growing complexity of organization, to the systems acquiring new properties and functions. This approach to the study of organization forms by moving from simple to complex and characterized by studying of mechanisms of organization elements in the whole, their integration to the system, has got a name of integratism. The basic feature of it is the study of the ways of including, integration elements into the structure [1].

Accepting integratism as a strategy of the scientific research, we should define the main specific features of its methodology, intended for analysis of organization of elements into a system, their relations and hierarchy. What specific characteristics must have a method suitable for such systemic approach to complicated dynamic and hierarchically organized systems?

Most likely that the leading idea of this system approach should be the selection of an optimal set of tools having a limited number of characteristics, and the specific feature - dialectics of relationships between dynamic and static parameters of elements and the intimate character of their functionality. However, as no method can simultaneously give the answer to all mentioned requests, we came to the synthesis of the next successive logical structure:

1. Dividing the system into sets of "elementary" components. These components do not need to be system-forming because the criteria of system-forming might be established only after the study of relations of these components.

2. Analysis of the relations of these components in space and in time and selection of system-forming elements.

3. Study of the properties of system-forming elements.

4. Analysis of the relation of system-forming elements and selection of the system-forming relations.

5. Study of the hierarchy of elements and their relations.

6. Description of the structure of the system (model) and its properties (forecast).

3 Factor analysis as a realization of the system approach

One of the most important problems of the methodology of the system approach is that of selection or creating an apparatus able to analyze the new type problems by adequate logical tools.

Let us consider factor logic as a principle of analysis. Suppose that elements of a system can be observed or measured on any finite and unique set for the whole system, for example, on the time axis or/and on the set of some homogeneous objects. This request to the set of realizations meets the main principle of organicism. Let us choose as many elements as we can, except elements with a clear functional dependence with already selected ones. This set of components in the factor analysis got the name of the matrix of individuals. In general, the system can be realized on more then one set of realizations representing the new specific aspect of the factor analysis. In this case the investigation of the system leads us to the analysis of the number of sets of realizations, which gives us an n-dimension matrix of observations.

After the components have been chosen and the matrix has been set, the matrix of correlations (in general case - n matrices) between parameters can be calculated. Factor analysis transforms this matrix to the matrix of factors, where each of them reflects a set of components connected to a one system-forming element and represents a system-forming connection of elements. It is important to note that by using the technic of principal components all factors become orthogonal and caused by different properties of the system.

Hence, we can see that the factor analysis follows the logic of the above mentioned theoretical ideas and their principles. The way of dismembering the system into a set of components meets the ideas of reductionism, and the way of creating the functionally caused structure of the system by the realizations of its components follows the basic principle of itegratism.

Thurstone [4, 5] and Harman [7, 8] considered the factor analysis as a statistical method for the compression of information, economic description of the data. At the same time, the goal of factor analysis consists in creating and investigation of models, conceptions and ideas, allowing one to analyze and interprete sets of experimental or observed data independent of their physical nature. Therefore, the goal of factor analysis consists in revealing functional units, forming the base of the change of variables.

An alternative to this view on the nature of factor analysis is represented by the conception, known as operationalistic or nominalistic [9–11]. The supporters of this conception insist that factors are only mathematical abstractions not corresponding to any realities, and therefore must be considered only as categories for classification of the elements but not as system properties.

Ahmavaara [12, 13] has especially emphasized that the factor model lies outside the sphere of statistical analysis, being founded on the arguments of more common nature illustrating this concept by the example of using the differential calculus in physics. The differential calculus has its own principles and methods, independent of the principles of statistics, so that a researcher appeals to the apparatus of statistics only if he wants to evaluate coefficients of the differential equation. The factor analysis also has not been considered only as a method of statistics, because the factor logic has its own principles and laws independent of any laws and principles of statistics. The factor logic has to be considered as a concretization of the common science logic applicable to the sciences, which regards the organization of the system as an object of their investigation, and which can examine elements of the system as a set of n-dimensional matrices. Therefore, factor analysis can be considered as a kind of a new form of mathematical language for a consistent description of the processes of dynamic structural-functional organization of complex systems in the environment dependent on statistical variations.

4 History of factor analysis and factors interpretation

A hundred years ago, Charles Spearman [14] published in the American Journal of Psychology the paper about the factor of general intelligence, based on fulfillment of all tests connected with intellectual functions. But until the middle of thirties, interpretation of the factors caused practically no problems since the mathematical technique of factor analysis, using in these years, assumed the existence of only one [14], two [7] or several [9] factors with *a priory* known structure. The main goal of the factor analysis was the control of conformity of a priory given factor structure to the experimental data, and the analysis of quantitative differences between tests. Only the multiple-factor analysis, proposed in 1935 by Thurstone [4], has allowed to pick out factors not defined *a priory*. However, all these methods of multiple-factor analysis, the centroid method [4], and the method of maximum likelihood [15], very cumbersome in calculations and, mainly, leading to different factor structures, caused the wave of disappointments in the factor analysis, especially in attempts of pithy interpretation of the factors, and therefore stipulated the domination of positivistic, operationalistic ideas in it. Failures in the interpretation led even such "functionalist" as Thurstone to the view on the factor analysis only as a scientific method for the confirmation or rejection of hypotheses, concerned the nature of the processes [5]. Rotation of the factor matrix allowed overcoming the uncertainty of interpretation, but criteria of the rotation itself were based on the very vague signs of "simple structure", or on the agreement with data obtained by other methods, other investigators and on the agreement to common principles of the concrete science.

Around 1950 the reputation of factor analysis suffered from overpromotion by a few overenthusiastic partisans. In retrospect, there were three things wrong with the way some people were thinking about factor analysis at that time. First, some people seemed to see factor analysis as *the* statistical method rather than *a* statistical method. Second, they were thinking in absolute terms about problems, for which a heuristic approach would have been more appropriate. Third, they were thinking of overly broad sets of variables ("we want to understand all of human personality", rather than "we want to understand the nature of curiosity"). Thus in three different ways, they were attempting to stretch factor analysis farther than it was capable of going [6].

The exit of this deadlock was shown by Hotelling proposed the method of principal components permitting calculation of the unique matrix of the orthogonal factors [16]. Although this method required many mathematical computations and could be used in practice only with a progress of computers, it immediately got an appreciation of many investigators. Thurstone first pointed that even the most powerful method of factor analysis - the centroid method - is not more then "calculating compromise" of the principal component method, which was later proved by Anderson and Rubin [17]. After the appearance of this method, factor analysis has got his second birth and has had a right to be considered as a method of the structure search in all fields of science.

Table 1 shows the distribution of papers concerning factor analysis in various fields of science and industry. The sample consists of 3460 papers hosted in the Internet by May 2004. Each paper has been marked according to its belonging to the certain type of these fields, sometimes more then one, so that the sum of numbers along a column is greater or equal to the figure in the last row. Although the distribution of papers in the Internet does not fully correspond to their real publications, one can see a lot of interesting tendencies, such as, for example, ongoing increase not only in absolute, but also in relative part of publications concerning various aspects of applications and development of factor analysis.

	1904 -1980	1981 -1985	1986 -1990	1991 -1995	1995 -2000	2000 -2004	Total
Biology	18	17	20	23	47	41	166
Chemistry	12	14	36	53	88	77	244
Chromatography	4	7	16	22	24	15	88
Ecology	2	4	11	15	61	45	138
Economics	14	12	9	4	20	26	85
Food	1	4	5	2	17	21	50
Geriatry	8	5	10	9	25	31	88
Image Processing	2	7	22	27	38	51	151
Industry	4	0	2	6	38	28	78
Magnetic Resonance	1	1	3	6	25	13	49
Medicine	30	32	64	67	109	116	418
Methodology	10	25	31	49	125	151	391
Operational Research	1	1	1	9	42	41	95
Physiology	20	26	38	39	51	29	203
Psychiatry	15	14	39	61	137	99	365
Psychology	93	86	159	219	379	344	1287
Spectroscopy	11	27	40	50	108	90	326
(a) Total FA-papers	196	242	408	545	1065	1002	3460
(b) All papers($*10^3$)	5186	1518	1890	2117	2430	1999	14707
(c) FA/All($*10^{-6}$)	38	159	216	257	438	501	235

Table 1. Distribution of papers on factor analysis in the Internet.

Data from the author's collection http://www.magniel.com/fa/data. Bottom rows:
(a) - total numbers of papers on factor analysis per the given time interval;
(b) - total number of publications from http://highwire.stanford.edu/;
(c) - ratio of papers on factor analysis to all papers in the Internet ($c=a/b\times10^{-6}$).

Factor analysis has been developed in two directions. First of all, serious efforts were undertaken towards the development of its mathematical technique, error estimation, special methods for analysis ordinal and nonlinear data.

However, from the methodological point of view, it is much more interesting to consider completely new objects of statistical investigation. As we know, "classical" factor analysis is using to study a two-dimensional matrix composed by correlation coefficients of individuals on the set of observations.

Exploratory factor analysis is directed to uncover the underlying structure of a system. The researcher's *a priori* assumption is that any variable may be associated with any factor. This is the most common form of factor analysis. There is no prior theory and one uses factor loadings to intuit the factor structure of the data. The results of an exploratory factor analysis may have heuristic and suggestive value and may generate hypotheses which are capable of more objective testing by other multivariate methods [18].

Confirmatory factor analysis allows us to test specific hypotheses regarding the factor structure and compare factor structures across the samples. The researcher's *a priori* assumption is that each factor (the number and labels of which may be specified *a priori*) is associated with a specified subset of indicator variables [19–22]. Confirmatory factor analysis as a method of latent variable modeling, together with a path analysis leading to structural equation modeling [23–25].

Correspondence analysis designed to analyze two-way and multi-way tables containing some measure of correspondence between the rows and columns. The results provide information which is similar in nature to that produced by factor analysis techniques, and they allow one to explore the structure of categorical variables included in the table [26].

If researcher is interested in the dynamics of observations on the time-scale, he can use dynamic factor analysis, based on shifted matrices of observations (see, for example, our works dealing with analysis of multi-unit activity of human brain [27–29]). Stochastic observed process is decomposed into a structured part (latent process) and a remainder (noise) [30–33]. Methodology and technique of shifted factor analysis is described in details in papers of Hong and Harshman [34–36]. Such technique can be used not only for analysis of time series, but for analysis of spacial series, for example, in image processing in physiology (see papers of Barber [37–40]. Factor analysis as a tool for pattern recognition is widely used, for example, in face identification [41–45].

Three-dimensional matrix of observations can be graphically represented and analysed by various methods of biplot-analysis [46–49].

Multiple (n-dimensional) factor analysis [50–55] is used in studying the multi-parametrical structure of complex system.

Hierarchical factor analysis is intended for studying factor hierarchy. It was proposed by Thompson [22] and popularized in the detailed discussions by Wherry [57]. In this strategy, a user first identifies clusters of items and rotate axes through those clusters; next the correlations between those (oblique) factors are computed, and then the correlation matrix of oblique factors is further factor-analyzed to yield a set of orthogonal factors that divide the variability in the items into that due to shared or common variance (secondary factors), and unique variance due to the clusters of similar variables (items) in the analysis (primary factors). Admirers of classical factor analysis use this technique also in our days [58, 59].

It was curious to factorize coefficients of information instead of correlation. Such attempts were made by us [60–62] and by Browne [63].

The logic of neural networks coincides with logic of factor analysis in the algorithms of neural networks factor analysis [64–76].

Without any doubt, it should not necessary lead to the discovery of "fundamental" or "basic" categories in the concrete field of science: according to the probabilistic principle of organization of complex systems, the factor structure might be completely random by the character of realization of its elements. Moreover, in the result of factor analysis one can come to pure mathematical abstractions. But at the same time the factor analysis has become more and more popular as a method oriented not only to the proving or rejecting the existing hypotheses, but also to the revealing of a new system of pithy components, promoting thereby a deeper understanding of the causality and the structural-functional organization of the system.

In this aspect, it is interesting to trace the evolution in views to the nature of factor analysis of such distinguished neurophysiologist (but not a specialist in factor analysis) as E.R. John. While in his early papers [77], the author, applying the method of main factors to the analysis of evoked potentials, tried to check the similarity of some processes considering factor analysis only as a method of classification having no physiological sense, in his further papers the method of principal components has been used for economic and precise quantitative description and mathematical identification of brain states applied to classifications of drugs [78].

Modern science is challenging existing methods of data analysis and interpretation towards automation of all steps of the scientific process. Let us consider, for example, *Virtual Scientist* - a set of computational procedures automating the method of inductive inference to derive a theory from observational data dominated by nonlinear regularities. Some of these procedures utilize SINBAD - a novel computational method of nonlinear factor analysis based on the principle of maximization of mutual information among non-overlapping sources, yielding higher-order features of the data which reveal hidden causal factors controlling the observed phenomena. The procedures build a theory of the studied subject by finding inferentially useful hidden factors, discovering interdependencies among its variables, reconstructing its functional organization, and describing it by a concise graph of inferential relations among its variables. The graph is a quantitative model of the studied subject, capable of performing elaborate deductive inferences and explaining behaviors of the observed variables by behaviors of other variables and discovered hidden factors [79].

Nevertheless, using the method of principal components for the revealing fundamental, significant and eventually system-forming elements (factors), especially in the absence of correlation between them, requests a specific care in the interpretation and scientific proof [80–82].

Under a contemporary tendency of dissemination of the sphere of factor analysis applications, the most serious attention must be paid to the interpretation of results, especially in analyzing the reasons causing the interrelations of components, having in mind goals and problems of the investigation in order to estimate their correspondence with the obtained factor model [83–86].

One should not, however, forget that factor analysis does not always give a possibility of the pithy interpretation of factors. The interpretation must be based on the data of a nature and properties of elements of the system, obtained by other methods. Factor analysis in this sense is only a link among the other stages of investigation; the connection with these links must be always maintained, and only the whole chain can lead to the solution of a problem [87–89]. Only the breadth of erudition of researchers and knowledge of principles of the functional integration of investigated systems are able to create a necessary basis for the objective interpretation of revealing factors.

References

[1] V.A. Engelgardt, Dokl. Biol. Sci. **6**, 799-822 (1970).

[2] H. Verhoog, In: *Zit er Toekomst in ons DNA*, Ed.: J. van der Wal and E. Lammerts van Bueren, p.17 (Rapport Werkgroep Genenmanipulatie en Oordeelsvormingmm, Driebergen, 1993); H. Verhoog, In: *Genmanipulation an Pflanze, Tier und Mensch - Grundlagen zur Urteilsbildung* p. 11 (Verlag Freies Geistesleben, Stuttgart, 1994).

[3] I.V. Blauberg, V.N. Sadovsky, and E.G. Yudin, *Systems Theory: Philosophical and Methodological Problems* (Progress, N.Y., 1977).

[4] L.L. Thurstone, *Vectors of the Mind* (U. of Chicago Press, Chicago, 1935).

[5] L.L. Thurstone, *Multiple-Factor Analysis* (U. of Chicago Press, Chicago, 1947).

[6] R.B. Darlington, *Regression and Linear Models* (McGraw-Hill, N.Y., 1990).

[7] K.J. Holzinger and H.H. Harman, *Factor analysis: A Synthesis of Factorial Methods* (U. of Chicago Press, Chicago, 1941).

[8] H.H. Harman, *Modern Factor Analysis* (U. of Chicago Press, Chicago, 1960).

[9] C. Burt, *The Factors of the Mind: An Introduction to Factor Analysis in Psychology* (U. of London Press, London, 1940).

[10] P.E. Vernon, *The Structure of Human Abilities* (Wiley, London, 1950).

[11] J. Elmgren, Goteborgs Universitets Arsskrift **64**, 5 (1958).

[12] Y. Ahmavaara, *On the unified factor theory of mind*, Ann. Acad. Sci. Fenn. B **106**, 1 (1957).

[13] Y. Ahmavaara and T. Markkanen, *The Unified Factor Model* (The Finnish Foundation for Alcohol Studies, Helsinki, 1958).

[14] C. Spearman, Am. J. Psychol. **25**, 201 (1904).

[15] D.N. Lawley, Proc. Roy. Soc. Edinburgh A **60**, 82 (1940).

[16] H. Hotelling, J. Educ. Psychol. **24**, 417 (1955).

[17] T.W. Anderson and H. Rubin, In: Proc. Third Berkeley Symp. Math. Stat. Probab., Ed. J. Neyman, vol.**5**, p.111 (U. of California Press, Berkeley, CA, 1956).

[18] K. Jöreskog, In: *Factor Analysis in 100*, Conf. U. North Carolina, Chapel Hill, NC, May 13-15, 2004.

[19] G.D. Garson, In: *QASS Factor Analysis Monographs: A Computer-Focused Review*, a review essay. Social Science Computer Review, **17**, p.129 (Spring, 1999).

[20] J. Scott Long, *Confirmatory Factor Analysis: A Preface to LISREL*, Quantitative Applications in the Social Sciences, vol. **33** (Sage Publications, Beverly Hills, CA, 1983).

[21] K. Jöreskog and D. Sörbom, *LISREL 8: Users Reference Guide* (Lawrence Erlbaum Associates, Hillsdale, NJ, 1996).

[22] B. Thompson, *Exploratory and Confirmatory Factor Analysis: Understanding Concepts and Applications* (American Psychological Association, 2004).

[23] K. Jöreskog and D. Sörbom, *Advances in factor analysis and structural equation models* (Abt Books, Cambridge, MA, 1979).

[24] D.J. Bartholomew, *Latent Variable Models and Factor Analysis* (Oxford U. Press, N.Y., 1987).

[25] J.C. Loehlin, *Latent Variable Models: An Introduction to Factor, Path, and Structural Analysis* (Lawrence Erlbaum Associates, Hillsdale, NJ, 1987).

[26] M. Greenacre, *Correspondence Analysis in Practice* (Academic Press, London, 1993).

[27] N.P. Bekhtereva, P.V. Bundzen, A.S. Kaplunovsky, and Y.K. Matveev, Sechenov Physiol. Journ. USSR **57**, 1605 (1971).

[28] P.V. Bundzen, Yu.L. Gogolitsin, E. David, A.S. Kaplunovsky, and P.D. Perepelkin, Sechenov Physiol. Journ. USSR **59**, 1803 (1973).

[29] P.V. Bundzen, Yu.L. Gogolitsin, and A.S. Kaplunovsky, In: *Biological Diagnosis of Brain Disorders*, Ed. S. Bogoch, p. 275 (Spectrum, N.Y., 1974).

[30] C. Heij and W. Scherrer, SIAM J. Control Optim. **35**, 1924 (1997).

[31] X. Giannakopoulos and H. Valpola, AIP Conf. Proc. **568**, 305 (2001).

[32] A.F. Zuur, R.J. Fryer, I.T. Jolliffe, R. Dekker, and J.J. Beukema, Environmetrics **14**, 665 (2003).

[33] A.F. Zuur, I.D. Tuck, and N. Bailey, Canad. J. Fisher. Aquat. Sci. **60** 542 (2003).

[34] R.A. Harshman, S. Hong, and M.E. Lundy, J. Chemom. **17**, 363 (2003).

[35] S. Hong and R.A. Harshman, J. Chemom. **17**, 379 (2003).

[36] S. Hong and R.A. Harshman, J. Chemom. **17**, 389 (2003).

[37] D.C. Barber, Phys. Med. Biol. **21**, 792 (1976).

[38] D.C. Barber, Phys. Med. Biol. **25**, 283 (1980).

[39] K.S. Nijran and D.C. Barber, Phys. Med. Biol. **30**, 1315 (1986).

[40] K.S. Nijran and D.C. Barber, Phys. Med. Biol. **31**, 1107 (1986).

[41] P.J. Hancock, A.M. Burton, and V. Bruce, Mem. Cognition **24**, 21 (1996).

[42] H. Moon and P.J. Phillips, Perception **30**, 303 (2001).

[43] A. Pujol, J. Vitriá, F. Lumbreras, and J.J. Villanueva, Pattern Recogn. Lett. **22**, 769 (2001).

[44] B.A. Draper, K. Baek, M.S. Bartlett, and J.R. Beveridge, Comput. Vis. Image Und. **91**, 115 (2003).

[45] R. Gottumukkal and V.K. Asari, Pattern Recogn. Lett. **25**, 429 (2004).

[46] K.R. Gabriel, Biometrika **59**, 455 (1971).

[47] A.S. Kaplunovsky and M.M. Bogoslovsky, Sechenov Physiol. Journ. USSR **59**, 1291 (1973).

[48] J. Graffelman and T. Aluja-Banet, Biometrical J. **45**, 491 (2003).

[49] M.R. Alves and M.B. Oliveira, J. Chemom. **17**, 594 (2003).

[50] T. Wansbeek and J. Verhees, Linear Algebra Appl. **127**, 631 (1990).

[51] Y. Zeng and P.K. Hopke, Chemom. Intell. Lab. Syst. **7**, 237 (1990).

[52] B. Escofier and J. Pagés, Computat. Stat. Data An. **18**, 121 (1994).

[53] G. Becker, Psychol. Methods **1**, 341 (1996).

[54] B.M. Wise, N.B. Gallagher, S.W. Butler, D.D. White Jr., and G.G. Barna, J. Chemom. **13**, 379 (1999).

[55] M.E. Timmerman and H.A. Kiers, Brit. J. Math. Stat. Psy. **53**, 1 (2000).

[56] G.H. Thomson, *The Factorial Analysis of Human Ability* (London U. Press, London, 1951).

[57] R.J. Wherry, *Contributions to correlational analysis* (Academic Press, London, 1984).

[58] J.A. Westerhuis, T. Kourti, and J.F. MacGregor, J. Chemom. **12**, 301 (1998).

[59] S. Le Dien and J. Pagés, Food Qual. Prefer. **14**, 397 (2003).

[60] A.S. Kaplunovsky, V.M. Klimenko and V.A. Grigoriev. In: Proc. Fifth Conf. Neurocybernetics, p.131 (Rostov-na-Donu, 1973).

[61] V.M. Klimenko and A.S. Kaplunovsky, Sechenov Physiol. Journ. USSR, **58**, 1484 (1972).

[62] V.M. Klimenko and A.S. Kaplunovsky, In: *Self-regulation of Neurophysiological Mechanisms of Integrative and Adaptive Brain Activity*, p.47 (Leningrad, 1972).

[63] M.W. Browne, Linear Algebra Appl. **70**, 51 (1985).

[64] J. Karhunen and J. Joutsensalo, Neural Networks **8**, 549 (1995).

[65] K.I. Diamantaras and S.Y. Kung, *Principal Component Neural Networks* (John Wiley & Sons, N.Y., 1996).

[66] D. Dong and T.J. McAvoy, Comput. Chem. Eng. **20**, 65 (1996).

[67] J-H. Jiang, J-H. Wang, C. Xia, and R-Q. Yu, Anal. Chim. Acta **336**, 209 (1996).

[68] L. Wang and J. Karhunen, Int. J. Neural Syst. **7**, 53 (1996).

[69] C. Fyfe, Neural Process. Lett. **6**, 33 (1997).

[70] C. Linneberg and A. Höskuldsson, J. Chemom. **14**, 573 (2000).

[71] S. Nicole, Computat. Stat. Data An. **33**, 425 (2000).

[72] E.S. Tzafestas, A. Nikolaidou, and S.G. Tzafestas, Math. Comput. Simulat. **51**, 145 (2000).

[73] R. Cancelliere and M. Gai, Nonlinear Anal. **47**, 5785 (2001).

[74] W.W. Hsieh, Tellus A **53**, 599 (2001).

[75] M. Tanaka, In: *Artificial Neural Networks* , Eds: G. Dorffner, H. Bischof, K. Hornik, Lect. Notes Comput. Sc. **2130**, 183 (2001).

[76] Y. Takahata, M.C. Costa, and A.C. Gaudio, J. Chem. Inf. Comp. Sci. **43**, 540 (2003).

[77] D.S. Ruchkin, J. Villegas, and E.R. John, Ann. N.Y. Acad. Sci. **115**, 799 (1964).

[78] E.R. John, P. Walker, D. Cawood, M. Rush, and J. Gehrmann, EEG Clin. Neurophysiol. **54**, 55 (1975).

[79] O. Kursun and O.V. Favorov, Natural Computing **3**, 207 (2004).

[80] J.E. Overall and C.M. Williams, Behav. Sci. **6**, 154 (1961).

[81] K. Buss and H. Buss, Jaunakais Mezsaimnieciba **13**, 5 (1970).

[82] D.J. Bartholomew, F. Steele, I. Moustaki, and J.I. Galbraith, *The Analysis and Interpretation of Multivariate Data for Social Scientists* (Chapman & Hall/CRC, Boca Raton, FL, 2002).

[83] V.P. Järveläinen, Silva Fenn. **5**, 281 (1971).

[84] P. Klemmer, Raumforsch. u. Raumordn. **29**, 6 (1971).

[85] M. Forni and M. Lippi, *The Generalized Dynamic Factor Model: Representation Theory* (Mimeo, Universita di Modena, 2000).

[86] M. Forni, M. Hallin, M. Lippi, and L. Reichlin, Rev. Econ. Stat. **82**, 540 (2000).

[87] M. Reuchlin *Les Méthodes Quantitatives en Psychologie* (PUF, Paris, 1962).

[88] M. Reuchlin, *Totalités, Éléments, Structures en Psychologie* (PUF, Paris, 1995).

[89] M. Reuchlin, Psychol. Histoire **4**, 31 (2003).