ELECTRON SPECTRUM IN NUCLEAR SPIN POLARIZATION INDUCED PERIODIC STRUCTURES

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Abstract. The novel meso-nucleo-spinics effect of the nuclear spin polarization induced periodic structure creation in a low-dimensional electron system is studied theoretically. It is shown that the periodically distributed nuclear magnetization results in the periodic hyperfine magnetic field which, in turn, creates a periodic electron structure. The electron wave functions and energy spectrum for such a structure are evaluated.

1. Introduction

Recent advances of semiconductor device miniaturization strengthen the need to take into account the influence of quantum mechanical effects on device performance since the statistical behavior of a quantum system may result in unpredictable device parameter fluctuations. These problems stimulate the search for alternative schemes for electronics and, at the same time the need to take into account the non-classical behavior of electrons. The possibilities of controlling the electron and nuclear spin instead of the electric charge give rise to the so-called spintronics[1]. The coherence of quantum states plays a key role in the quantum state temporal evolution. Coherent approaches to electronics and spintronics possess two major advantages[1].

1. Interference between two coherently occupied quantum states separated by the energy ΔE can result in rapid oscillations of a spin magnetization orientation with a frequency $\nu = \Delta E/h$ (here h is the Planck constant) driving the operation of ultrafast devices and permitting to tune the device operating frequency. Magnetic field can control electron spin precession frequencies in semiconductors at a rate of tens of gigahertz per tesla.

2. Interference may be used in quantum computation where nuclear or electron spins isolated from their environment play the role of computational quantum bits (qubits).

The electron spin qubits, however, have an essential disadvantage from the viewpoint of quantum computation applications. The electron spin polarization rapidly vanishes after the controlling signal is switched off, and for this reason the information expressed in electron qubits cannot be stored for a sufficiently long time. Nuclear spins, on the contrary, possess a sufficiently large relaxation time T_1, up to several hours at low temperature[2]and can therefore be considered as the best suitable candidates for a qubit [3]-[5]. The emerging problem of nuclear spins manipulation and their inhomogeneous spatial distribution is of great importance. In the absence of an external magnetic field, the hyperfine magnetic field B_{hf} caused by the magnetization of the highly polarized nuclear spins can produce a Zeeman splitting of the conduction electron energy spectrum equivalent to the one corresponding to an external field of several Teslas. A new class of phenomena related to the electron behavior in mesoscopic systems under the influence of a sufficiently strong hyperfine magnetic field due to the polarized nuclei is called meso-nucleo-spinics[6]. The hyperfine interaction is believed to play a central role in the possible solid state realizations of future quantum computation devices. The main ingredients of such a prototype system are nuclear spins, or qubits, coupled through the hyperfine interaction to a phase coherent electron spin system which can exist in a two-dimensional electron gas (2DEG) in a doped heterostructure [7].

Recently the modification of electron wave functions and energy spectrum in the inhomogeneous hyperfine magnetic field has been investigated theoretically for the case of a ring[8], quantum dot[9], and quantum wire[10]. It has been shown that the hyperfine field induced by polarized nuclei results in the electron spin polarization, leading to energy splitting and spatial confinement of electrons[8]-[10]. In fact, the action of the spatially inhomogeneous hyperfine field has some similarities with the combination of the external magnetic field and a periodic external electrical [11] or magnetic [12] potential.

In this paper we investigate theoretically the *meso-nucleo-spinics* effects in the case of a laterally periodic hyperfine field. The main physical difference from the previously studied cases is in the creation of a laterally periodic electron Zeeman splitting even in the absence of an external magnetic field. The mechanism of the chain of events can be described qualitatively as follows. The free electron spin system can be optically polarized by an elliptically polarized light wave [2]. A spatially inhomogeneous distribution of free electrons, or a dynamic grating, can be created by two interfering laser beams[13]. In such a case, the optically induced magnetization of the free electron system would also be spatially inhomogeneous imitating the grating configuration. Optical electron spin excitation[2] and polarized electron spin transport[14] in semiconductor nano-structures are strongly

coupled to the nuclear spin subsystem. The magnetization can be transferred from electrons to nuclei due to the hyperfine interaction mechanism. The electron spin polarization rapidly vanishes after the removal of the external radiation while the spatially inhomogeneous magnetization of the nuclei can be stored for a sufficiently large relaxation time T_1. This process may represent a writing and storage of information. The reading of the stored information can be realized through the creation of a free electron dynamic grating, but this time with a linearly polarized light. The dynamic electron grating would receive the spatially inhomogeneous magnetization from the polarized nuclei very rapidly through the same hyperfine interaction process and it would modulate the incident light beam. We evaluated the light induced hyperfine magnetic field. We solved in a closed form the Schroedinger equation for an electron subjected to such a field. Wave functions expressed in terms of Mathieu functions and a discrete electron energy spectrum consisting of paths have been obtained.
The paper is constructed as follows. The hyperfine field is calculated in the second section. The Schroedinger equation is analyzed in the third section. The conclusions are presented in the fourth section.

2. The Light Induced Hyperfine Magnetic Field

Consider a 2DEG situated in a GaAs/AlGaAs heterostructure. We assume that the spins of the 2DEG are optically polarized with the light intensity dynamic grating[13]

$$I = I_0 \left[1 + m \cos\left(ky - \Omega t\right)\right] \tag{1}$$

where I, I_0 are the light intensity and its amplitude, m is the modulation coefficient, k, Ω are the wave vector and frequency difference of the two interfering laser beams, respectively.
The temporal evolution of the hyperfine field B_{hf} is governed by two main mechanisms: the nuclear-spin relaxation determined by the relaxation time T_1, and the nuclear-spin diffusion determined by the spin-diffusion coefficient D. Assuming that the hyperfine field B_{hf} does not depend on the coordinates x, z we write for $B_{hf}\left(y, t\right)$ the one-dimensional diffusion equation[15]

$$\frac{\partial B_{hf}}{\partial t} = D \frac{\partial^2 B_{hf}}{\partial y^2} - \frac{B_{hf}}{T_1} \tag{2}$$

Optical spin polarization methods permit the creation of one-dimensional structures of the order of magnitude of $1 \mu m$[16]-[19]. The initial condition for the hyperfine field at $t = 0$ are assumed to be periodic in accordance with the form of the light grating (1)

$$B_{hf}\left(y\right) = B_0 \left[1 + m \cos ky\right] \tag{3}$$

Then the solution of (2) is sought to be

$$B_{hf}(y,t) = Y(y)\,T(t)\exp\left(-\frac{t}{T_1}\right) \tag{4}$$

Substituting (4) into (2) and solving together with (3) we obtain

$$B_{hf}(y,t) = B_0\exp\left(-\frac{t}{T_1}\right)\left[1+(m\cos ky)\exp\left(-\frac{t}{\tau}\right)\right] \tag{5}$$

where the nuclear-spin diffusion time $\tau = \left(k^2 D\right)^{-1}$ which is rather long in semiconductors at low temperatures[17].

3. The Electron Energy Spectrum in a Periodic Hyperfine Field

The microscopic description of the electrons in the hyperfine magnetic field is based on the following Hamiltonian H

$$H = -\frac{\hbar^2}{2m^*}\Delta + V(x) + U(z) + \mu_B\sigma\cdot\mathbf{B}_{hf}(y,t) \tag{6}$$

where $\hbar = h/2\pi$, m^* is the electron effective mass, Δ is the Laplace operator, μ_B is the Bohr magneton, and σ is the electron spin. The potential $V(x)$ defines the electron transmission probability through the heterojunction. In our case it is chosen to be $V(x) = V_0 = const$ for $|x| \leq L/2$ and $V(x) = 0$ otherwise where L is the heterostructure width in the x direction. In such a situation the electron transmission probability $P = 1$ and the influence of $V(x)$ is negligible[10]. The dependence on the z coordinate can also be ignored under the typical assumption that the 2DEG is filling only the lowest subband corresponding to the confinement in the z direction[10].
Consider now the time dependence of the hyperfine field (5). The temporal scale of both the relaxation and the diffusion processes in the nuclear spin system is several orders of magnitude larger than the temporal scale of the processes in the electron system at or close to equilibrium as it was mentioned above[2]. For this reason, we suppose that the electrons are in the steady-state regime. Then, the time-independent Schroedinger equation for the electron wave function $\psi(y)$ with the Hamiltonian (6) takes the form

$$-\frac{\hbar^2}{2m^*}\frac{\partial^2\psi(y)}{\partial y^2} - \mu_B B_{hf}(y,t)\,\psi(y) = E\psi(y) \tag{7}$$

Here only the electrons with the spins opposite to the hyperfine field are taken into account since the effective potential is attractive for such electrons. Substituting the hyperfine field (5) into (7) we obtain

$$\frac{\partial^2\psi(y)}{\partial^2 y} + \frac{2m^*}{\hbar^2}\left[E + \mu_B B_0\exp\left(-\frac{t}{T_1}\right)\left[1+(m\cos ky)\exp\left(-\frac{t}{\tau}\right)\right]\right]\psi(y) = 0 \tag{8}$$

In GaAs/AlGaAs heterostructures a hyperfine field B_{hf} of several tesla can be achieved[20],[21] which results in the hyperfine energy splitting $\sim 10^{-4}eV$. Introducing the dimensionless variable $\theta = ky/2$ we obtain from equation (8)

$$\frac{\partial^2 \psi(\theta)}{\partial \theta^2} + \left[\widetilde{E} - 2q\cos 2\theta\right]\psi(\theta) = 0 \tag{9}$$

where

$$\widetilde{E} = \frac{8m^*}{k^2\hbar^2}\left[E + \mu_B B_0 \exp\left(-\frac{t}{T_1}\right)\right];$$

$$q = -\frac{4m^*\mu_B B_0 m}{k^2\hbar^2}\exp\left[-\left(\frac{1}{T_1} + \frac{1}{\tau}\right)t\right] \tag{10}$$

Note that usually the modulation depth of the dynamic grating is small $m \ll 1$. Numerical estimations show that for the typical values $m = 0.1$, $m^* = 0.067m_e$ and a maximum value of the hyperfine field $B_0 = 5$ tesla $|q| \sim 1$ where m_e is the free electron mass.

Equation (9) is a well-known Mathieu equation[22]. It has the solution

$$\psi_{\alpha,n}(\theta) = \exp\left[i(2n+\alpha)\theta\right]A_\nu(\theta),\ \nu = 2n+\alpha,\ n = \pm 1, \pm 2, ... \tag{11}$$

where the periodic with a period of π functions $A_\nu(\theta)$ can be represented by the convergent Fourier series

$$A_\nu(\theta) = a_0^{(\nu)} + \sum_{l=-\infty}^{\infty} a_l^{(\nu)} \exp(2l\theta i) \tag{12}$$

The corresponding energy spectrum can be found from the dispersion equation [22]

$$\sin^2\frac{\pi v}{2} = \Delta(0)\sin^2\frac{\pi\sqrt{\widetilde{E}}}{2} \tag{13}$$

where $\Delta(0)$ is the infinite determinant which is too involved and it is not presented here. The functions $\psi_{\alpha,n}(\theta)$ are real and periodic with a period π for even n (whole-period solutions), or 2π for odd n (half-period solutions) and constitute an orthogonal system when α is an integer. Then the functions (11) are the Floquet solutions. There exist even and odd orthogonal eigenfunctions $ce_n(\theta)$, $se_n(\theta)$, and the eigenvalues $\widetilde{E}_n$ denoted a_n and b_n, for the even and odd solutions $ce_n(\theta)$, $se_n(\theta)$, respectively [22]. The solutions are normalized:

$$\int_0^{2\pi} \left[ce_n(\theta)\right]^2 d\theta = \int_0^{2\pi} \left[se_n(\theta)\right]^2 d\theta = \pi \tag{14}$$

The periodic functions $ce_n(\theta)$, $se_n(\theta)$ can be expanded in the Fourier series.

$$ce_{2n}(\theta,q)=\sum_{r=0}^{\infty}A_{2r}\cos(2r\theta)\,;$$

$$ce_{2n+1}(\theta,q)=\sum_{r=0}^{\infty}A_{2r+1}\cos[(2r+1)\theta] \tag{15}$$

and

$$se_{2n+2}(\theta,q)=\sum_{r=0}^{\infty}B_{2r+2}\sin[(2r+2)\theta]\,;$$

$$se_{2n+1}(\theta,q)=\sum_{r=0}^{\infty}B_{2r+1}\sin[(2r+1)\theta] \tag{16}$$

where the coefficients $A_{2r}, A_{2r+1}, B_{2r+2}, B_{2r+1}$ depend on q. The eigenvalues $q\left(\widetilde{E}\right)$ define the boundary lines between the stable and unstable regions in the $\left(q,\widetilde{E}\right)$ plane. In our case $(q<0)$ the relations between the eigenvalues are following

$$a_0<a_1<b_1<b_2<a_2<a_3<b_3<b_{4\ldots} \tag{17}$$

and the stable regions correspond to the conditions

$$a_{2n}\le\widetilde{E}_{2n}\le a_{2n+1}, b_{2n+1}\le\widetilde{E}_{2n+1}\le b_{2n+2} \tag{18}$$

For large $n\to\infty$ $\widetilde{E}=n^2$, and the electron energy spectrum reduces to the levels in an infinite potential well. In the case of a weak hyperfine field $B_0\lesssim 0.5$ tesla the value of the parameter $|q|\ll 1$, and the asymptotic expansions of some first eigenvalues neglecting higher power terms have the form

$$a_0(q)\approx-\frac{q^2}{2};\; a_1(-q)=b_1(q)\approx 1-q \tag{19}$$

$$a_2(q)\approx 4+\frac{5q^2}{12}; b_2(q)\approx 4-\frac{q^2}{12}; a_3(-q)=b_3(q)\approx 9-\frac{q^2}{16} \tag{20}$$

The asymptotic expansions of the first eigenfunctions linear in q are following

$$ce_0(\theta)\approx\frac{1}{\sqrt{2}}-\frac{q}{2\sqrt{2}}\cos(2\theta)\,;\; ce_1(\theta)\approx\cos(\theta)-\frac{q}{8}\cos(3\theta) \tag{21}$$

$$se_1(\theta)\approx\sin(\theta)-\frac{q}{8}\sin(3\theta)\,;\; ce_2(\theta)\approx\cos(2\theta)-q\left(\frac{\cos(4\theta)}{12}-\frac{1}{4}\right) \tag{22}$$

$$se_2(\theta)\approx\sin(2\theta)-q\frac{\sin(4\theta)}{12} \tag{23}$$

The dependence of the ground state wave functions ce_0 on time and spatial periodicity is graphically illustrated in Fig.1.

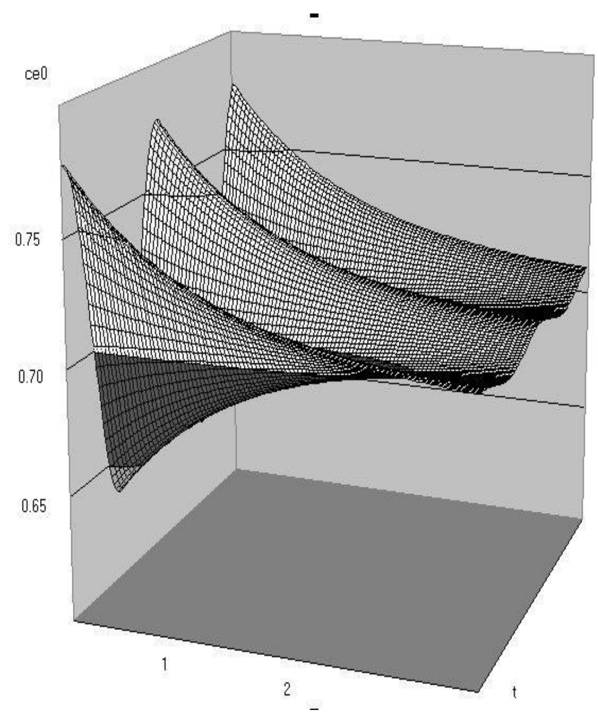

Figure 1. The time dependence of the ground state wave function ce_0, at fixed magnetic field $\widetilde{B} \equiv \frac{4m^* \mu_B B_0 m}{k^2 \hbar^2} = 0.1$, here $t \mapsto \left(\frac{1}{T_1} + \frac{1}{\tau}\right) t$, see text .

4. Conclusion

The novel *meso-nucleo-spinics* effect of the electron periodic structure creation under the hyperfine magnetic field is predicted theoretically. The periodic dynamic grating of two circularly polarized interfering laser beams creates a periodic distribution of polarized electron spin magnetization. This magnetization polarizes the nuclei spins due to the electron nuclear spin interaction. The relaxation time of the electron system is very small, and the electron spin magnetization rapidly disappears after the light grating is switched off. The spatially periodic nuclear spin magnetization is stored for a sufficiently large relaxation time T_1. The hyperfine magnetic field caused by this magnetization is spatially periodic and can be strong enough, up to several tesla. The electron spin interaction energy in such a field plays the role of a confining potential. The Schroedinger equation (8) reduces to the Mathieu equation which has stable periodic solutions expressed in terms of the even and odd Mathieu functions $ce_n(\theta)$, $se_n(\theta)$ for a set of energy bands, or stable regions separated by unstable regions. The boundaries of these bands are determined by the discrete series of eigenvalues $\widetilde{E}_n$. The explicit expressions for the eigenfunctions and eigenvalues in the case of a weak hyperfine field are presented. These results clearly show that information can be written, stored for a sufficiently long time and read by using the polarized nuclear spin system.

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