Roles of the surface 
and the bulk electronic states 
in the de Haas - van Alphen oscillations 
of two-dimentional electron gas

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Abstract

Using the Bloch approach applied to a two-dimensional metal bar we find that the Peierls result for zero-temperature magnetization of spinless electron gas consists of sum of two magnetic moments originating of the bulk and the surface electronic states.

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The theory of the two-dimensional de Haas-van Alphen effect was started by Peierls [1]. In the zero-temperature limit he obtained that the magnetization in an ideal two-dimensional electron gas has a sharp, saw-tooth form and a constant, with field, amplitude. This behavior is typical for the 2D electron system with fixed number $N$ of particles per unit area.

In another situation when some reservoir could make variation of $N$ possible such that the system chemical potential keeps a constant value the magnetization has sharp oscillations of so called inverse saw-tooth form [2].

The important consequence of the sharp saw-tooth shape of magnetization oscillations is an extremely slow convergence of its Fourier representation. It means practical useless of application of Poisson summation formula, so successful in 3D case. It is more convenient, therefore, to perform
explicitly the summation over the Landau levels, since only two, adjacent to
the Fermi energy, Landau levels are partially full. This approach has been
developed in series of papers by I. Vagner and co-authors [3]. They have
obtained an analytical expression for the magnetic field dependence of the
chemical potential and magnetization as well for the envelope of the magnet-
tization oscillations at finite temperature in the limit of sharp Landau levels.
The problem is simplified in presence of impurities then the oscillations are
smoothed and the Lifshitz-Kosevich approach to the 2D electron gas, but
taking into account the chemical potential oscillations, yields the reasonable
results at any temperature including zero temperature limit [4].

Usual treatment of electron gas magnetization is based on the calculation
of the partition function with summation over Landau levels with fixed de-
gree of degeneracy per unit area [5]. The Landau level energies are taken as
the degenerate eigenvalues of the Schrödinger equation in the bulk of metal.
The degeneracy of Landau states is lifted near the specimen boundary. So,
there are the two group electrons: (i) occupying the quantum states in the
bulk and (ii) in the surface of the metal. It is interesting to know what are
the roles these two group of electrons play in observable physical e-
eff
The more subtle derivation by F.Bloch [6] takes into account both the
electrons occupying the quantum states in the bulk and in the surface of
the metal [7] reproduces the Landau result for the electron gas diamagnetic
moment in the high temperature limit $T \gg h\omega_c$. This approach devel-
oped further in the paper by the present author [8] demonstrates that two
groups of electrons give equal contributions to the Landau diamagnetism.
On the other hand, it was concluded that only bulk electrons produce the
de Haas-van Alphen magnetization oscillations. Indeed, the oscillating part
of magnetization at $h\omega_c \ll \varepsilon_F$ given by the Eq. (21) in the paper [8] is
determined by the bulk states. However, it depends on the strongly oscillat-
ing with magnetic field chemical potential which is determined by equation
$N = -\partial\Omega/\partial\mu$ where both group of electronic states are important. More-
over, in the integer quantum Hall effect region when the distance between
Landau levels $h\omega_c$ is of the order of the Fermi energy $\varepsilon_F$ both groups of
electronic states give contributions to the oscillating magnetization. Hence
the conclusion that only bulk electrons produce the de Haas-van Alphen
magnetization oscillations is in fact incorrect.

In general, the division of bulk and surface electronic states contribu-
tions to the two dimensional electronic gas magnetic moment oscillations
is nontrivial problem. It is simplified in the zero-temperature limit where
one can perform explicitly the summation over the Landau levels following
to the papers by Peierls [1] and Schoenberg [2]. It is done in the present
We consider the two-dimensional (2D) clean metal bar with length $A$ in the $x$ direction and width $B$ in the $y$ direction ($|y| \leq B/2$) under a perpendicular magnetic field $\mathbf{H} = (0, 0, H)$. The thermodynamic potential of the electron gas is

$$
\Omega = -T \sum_\nu \ln \left( 1 + e^{\frac{\mu - \varepsilon_\nu}{T}} \right),
$$

where the electron energies $\varepsilon_\nu$ are determined as eigenvalues of the Schrödinger equation

$$
\left( \frac{\hbar^2}{2m} \left[ \left( -i \frac{\partial}{\partial x} - \frac{eHy}{\hbar c} \right)^2 - \frac{\partial^2}{\partial y^2} \right] + V(y) \right) \psi_\nu = \varepsilon_\nu \psi_\nu.
$$

The potential $V(y)$ is negligible everywhere except near the edges, where it increases up from zero at $|y| = B/2 - \Delta$ to infinity at $|y| = B/2$. The length $\Delta$ is chosen much larger than magnetic length $\lambda_H = \sqrt{\hbar c/eH}$:

$$
\lambda_H \ll \Delta \ll B
$$

The search of a solution in the standard form

$$
\psi_\nu = \exp(iqx) \varphi_\nu(y),
$$

where $q = 2\pi Q/A$ and $Q$ is an integer, leads us to the following eigenproblem

$$
\hat{H} \varphi_\nu = \varepsilon_\nu \varphi_\nu,
$$

$$
\hat{H} = \frac{\hbar^2}{2m} \left[ \left( q - \frac{eHy}{\hbar c} \right)^2 - \frac{\partial^2}{\partial y^2} \right] + V(y).
$$

It is clear that if the "equilibrium position" $y_0 = hqc/eH$ is limited by

$$
-\frac{B}{2} + \Delta \leq \frac{hqc}{eH} \leq \frac{B}{2} - \Delta,
$$

then to a quite good approximation the eigen functions of Eq. (5) are Landau wave functions

$$
\varphi_{nq}(y) = \frac{1}{\sqrt{2^n n! \sqrt{\pi \lambda_H}}} \exp\left[ -\left( y - y_0 \right)^2 / 2 \lambda_H^2 \right] H_n\left( \frac{y - y_0}{\lambda_H} \right)
$$

207
with \( q \)-independent eigenvalues

\[
\varepsilon_n = \hbar \omega_c \left( n + \frac{1}{2} \right),
\]

\( \omega_c = eH/m \). If \( y_0 \) is out the interval (6) then the eigenvalues \( \varepsilon_\nu = \varepsilon_{qn} \) are not so simple: they are \( q \) dependent and tend to infinity when \( |q| \to eHB/2\hbar c \).

Applying the standard notations for quantum mechanical averages \( \langle \hat{A}_n \rangle = \int dy \langle \varphi_\nu \hat{A} \varphi_\nu \rangle \) we obtain the following equality:

\[
-H \frac{\partial \varepsilon_{qn}}{\partial H} = q \frac{\partial \varepsilon_{qn}}{\partial q} - \frac{(eH)^2}{mc^2} ((y - y_0)^2)_\nu,
\]

where

\[
-H \frac{\partial \varepsilon_{qn}}{\partial H} = -\left< \frac{\partial \hat{H}}{\partial \hat{H}} \right> \nu = \langle \hat{M}_z \rangle \nu,
\]

and

\[
\frac{\partial \varepsilon_{qn}}{\partial q} = \left< \frac{\partial \hat{H}}{\partial q} \right> \nu = \hbar \langle \hat{v}_x \rangle \nu.
\]

For the equilibrium value of the system magnetic moment at given temperature we have

\[
M = -\left( \frac{\partial \Omega}{\partial H} \right)_\mu = -\sum_{nQ} \frac{\partial \varepsilon_{qn}}{\partial \Omega} e^{\varepsilon_{qn}/\mu} + 1 = -A \frac{eH/2\hbar c}{2\pi} \int dq \sum_{n=0}^{\infty} e^{\varepsilon_{qn}/\mu} + 1.
\]

The integral over \( q \) can be written as

\[
\int dq = \left\{ \begin{array}{c}
e^H(B/2-\Delta)/\hbar c \quad \int dq = \int_{-\infty}^{\infty} dq + \int_{-\infty}^{\infty} dq, \\
-e^H(B/2-\Delta)/\hbar c
\end{array} \right\}
\]

where the infinite limits are taken due to fast exponential convergency of the integral when \( \varepsilon_{qn} \to \infty \) for \( q \) outside the interval (6). Correspondingly the magnetic moment presents the sum of three terms

\[
M = M_1 + M_2 + M_3
\]

For the first term the energy levels have the \( q \) independent value (7); hence

\[
M_1 = -\frac{eS}{2\pi \hbar c} \sum_{n=0}^{\infty} e^{\varepsilon_n/\mu} + 1.
\]
where $S = AB$ is the bar area. For the second term, by making use the equality (8) and omitting the contribution proportional to $(y - y_0)^2$, which is of the order of $\Delta/B$ in comparison with other terms, we obtain

$$M_2 = \frac{A}{2\pi H} \int dq \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} e^{\frac{e(nq - \mu)}{\hbar c}} + 1$$

Taking into account that $M_2 = M_3$ finally we have

$$M = M_1 + 2M_2 = -\left( \frac{\partial \Omega}{\partial H} \right)_\mu,$$

where

$$\Omega = -\frac{eHST}{2\pi} \sum_{n=0}^{\infty} \ln \left( 1 + e^{\mu - \varepsilon_n} \right).$$

One can rewrite this result also as

$$M = M_1 + 2M_2 = \left( H \frac{\partial}{\partial H} + 1 \right) \left( -\frac{\Omega}{H} \right)_\mu$$

As follows from the derivation, the first term here

$$M_1 = -H \frac{\partial}{\partial H} \left( \frac{\Omega}{H} \right)_\mu$$

originates from the electrons occupying the Landau states situated in the bulk of metal. To work at the fixed number of particles one can rewrite $M_1$ as

$$M_1 = -\frac{eS}{2\pi \hbar c} \sum_{n=0}^{\infty} \varepsilon_n - H \frac{\partial \mu}{\partial H} \int_{-\infty}^{\infty} e^{\frac{e(nq - \mu)}{\hbar c}} + 1$$

$$= -H \frac{\partial}{\partial H} \left( \frac{\Omega}{H} \right)_\mu - N \frac{\partial \mu}{\partial H},$$

where

$$\Omega = -\frac{eHST}{2\pi} \sum_{n=0}^{\infty} \ln \left( 1 + e^{\mu - \varepsilon_n} \right).$$
where unlike to Eq.(19) the differentiation should be performed taking into account the chemical potential field dependence.

At $T = 0$ when $n$ is the number of Landau level occupied by electrons, such that $j = n - 1$ is amount of completely filled levels, the thermodynamic potential is [2]

$$
\Omega = \frac{1}{2} gn^2 \hbar \omega_c - g \mu n,
$$

(21)

where $g = S/2\pi \lambda_H^2 = NH/H_0$ is the degree of degeneracy.

Hence, for the bulk states moment, taking into account $\mu = \hbar \omega_c (n+1/2)$, we obtain

$$
M_1 = \frac{e \hbar}{2mc} \left[ gn(n+1) - N(2n+1) \right] = N \frac{e \hbar}{2mc} \left[ \frac{H}{H_0} (j+1)(j+2) - (2j+3) \right].
$$

(22)

The second term

$$
2M_2 = - \frac{\Omega}{H}
$$

(23)

is due to the electrons filling the edge states.

Hence, for the edge states moment we obtain

$$
2M_2 = \frac{e \hbar}{2mc} gn(n+1) = N \frac{e \hbar}{2mc} \frac{H}{H_0} (j+1)(j+2)
$$

(24)

The total magnetic moment

$$
M = M_1 + 2M_2 = N \frac{e \hbar}{2mc} \left[ 2\frac{H}{H_0} (j+1)(j+2) - (2j+3) \right]
$$

(25)

coincides with derived by Peierls [1].

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References


