HAIT Journal of Science and Engineering A, Volume 5, Issues 1-2, pp. 243-254

Double-dot transport in the spin blockade regime

Gloria Platero[1,*] and Jesús Iñarrea[1,2]

[1] *Instituto de Ciencia de Materiales, CSIC, Cantoblanco, Madrid 28049, Spain*
[2] *Escuela Politécnica Superior, Universidad Carlos III, Leganes, Madrid, Spain*
[*] *Corresponding author: gloria.platero@icmm.csic.es*

Received 13 November 2007

Abstract

We analyzed theoretically electronic transport through a double quantum dot in the spin blockade regime in the presence of hyperfine interaction. The electron and nuclei spin interaction produces electron spin flip which partially removes spin blockade. Induced nuclei spin polarization produces a finite magnetic field which acts on the electrons generating an additional Zeeman splitting of the electronic spin up and down levels in each quantum dot. This additional Zeeman splitting changes dynamically with the electronic levels occupation. Then, strong feed-back between the induced nuclear polarization within each quantum dot and the electronic charge distribution occurs and it produces strong non linearities in the current and in the nuclear polarization of each quantum dot as a function of magnetic field, such as hysteretic behavior. This bistable behavior is analyzed for different densities of nuclei within each quantum dot, i.e., for different intensities of the hyperfine interaction.

PACS: 73.63.Kv, 73.23.Hk, 72.10.Di

1 Introduction

Recent transport experiments in vertical double quantum dots (DQD's) show that Pauli exclusion principle is important[1] in current rectification. In particular, spin blockade (SB) is observed at certain regions of dc voltages. The interplay between Coulomb and SB can be used to block the current in one direction of bias while allowing it to flow in the opposite one. Then DQD's could behave as externally controllable spin-Coulomb rectifiers with potential application in spintronics as spin memories and transistors.

Spin de-coherence [2, 3] and relaxation processes induced, for instance, by spin-orbit (SO) scattering [4] or Hyperfine (HF) interaction [5], partially remove SB producing a leakage current.

Nuclear spin relaxation in two-dimensional electron gases (2DEG) has been theoretically investigated by Vagner and coworkers [6]. In particular, spin relaxation in a 2DEG in the Quantum Hall regime was investigated [7-9]. Also, Vagner *et al.* investigated the formation of a quantum wire by the hyperfine magnetic field induced by polarized nuclei acting on conduction electron spins [10]. Hyperfine interaction between nuclei and electron spins is a contact interaction where one electron flips its spin and one nuclei spin flips to the opposite polarization as the electron spin does. As Zeeman splitting for electrons is much larger than the one corresponding to the nuclei, the flip-flop process is inelastic and a dissipation through a phonon bath has to mediate it. A theoretical model for spin flip rate due to hyperfine interaction mediated by phonons in a 2DEG was proposed by Israel Vagner [11]. There, second order in time dependent perturbation theory is required because this process involves two steps: first both electron and nuclear spins are reversed via the hyperfine interaction and second, a phonon is absorbed by the spin system [11]. Later on, similar ideas were put forward for phonon-mediated hyperfine interaction in quantum dots [5].

In the present paper we deal with the analysis of transport through weakly coupled double quantum dots in the presence of an in-plane magnetic field. In particular, we have focussed our calculations for DC bias voltages within the SB region and we have analyzed the tunneling current as a function of the DC bias voltage and external magnetic field. The external magnetic field B produces an homogeneous Zeeman splitting within each quantum dot. We consider up to two extra electrons in the system. Transport calculations show that the current flows till SB is reached, it occurs once the electrons in the two dots have spins with the same polarization. Spin-flip through HF interaction partially removes SB. Spin flip rate is evaluated in our model by perturbation theory.

2 The model

The theoretical model presented in this paper has been developed in the frame of rate equations. We consider a hamiltonian: $H = H_L + H_R + H_T^{LR} + H_{leads} + H_T^{l,D}$ where $H_L(H_R)$ is the hamiltonian for the isolated left (right) QD and is modelled as one-level (two-level) Anderson impurity. $H_T^{LR}(H_T^{l,D})$ describes tunnelling between QD's (leads and QD's)) and H_{leads} is the leads hamiltonian. We consider a basis for up to two extra electrons in the system : the left dot (n_1) can have up to one electron and the right one (n_2) can fluctuate between one and two (intradot singlet state) keeping the sum ($n_1 + n_2$) between one and two. Our basis consists on twenty states but due to the voltages applied, those which mainly participate in the dynamics are:

$$\begin{aligned} &|1\rangle = |0,\uparrow\rangle; |2\rangle = |0,\downarrow\rangle; |3\rangle = |\uparrow,\uparrow\rangle; |4\rangle = |\downarrow,\downarrow\rangle; \\ &|5\rangle = |\uparrow,\downarrow\rangle; |6\rangle = |\downarrow,\uparrow\rangle; |7\rangle = |0,\uparrow\downarrow\rangle \end{aligned} \tag{1}$$

We solved the master equations for the occupations of the electronic states:

$$\dot{\rho}(t)_s = \sum_{m \neq s} W_{sm}\rho_m - \sum_{k \neq s} W_{ks}\rho_s \tag{2}$$

where $W_{i,j}$ is the transition rate state j to state i for the tunneling through the contact barriers and also for the spin flip scattering due to HF interaction. We have as well considered weak coupled quantum dots where the interdot tunneling is accounted for also by a scattering rate. Inter-dot transition rates in fact account for both elastic tunneling and inelastic phonon assisted tunneling. The corresponding expression for the elastic inter-dot tunneling is given by:

$$W_{1,2(2,1)} = \frac{T_{1,2}^2}{\hbar}\delta(\epsilon - (\mu_1 - \mu_2 + eV_{12})) \tag{3}$$

where $T_{1,2}$ is the transmission through the inner barrier and V_{12} is the voltage drop between the QD's.μ_1 and μ_2 are the chemical potentials of the left and right dots respectively, and V_{12} the voltage drop between the dots. For inelastic transitions, energy is exchanged with phonons in the environment. In other words, at $T \approx 0$ (we have considered zero temperature in our calculations) the inelastic tunneling between the two dots is assisted by the emission of acoustic phonons, yielding a significant contribution to the current. This contribution has been experimentally measured by Fujisawa *et al.* [2] and theoretically analyzed by Brandes *et al.* [12]. In order to calculate the inelastic transition rate $W_{1,2}^{ph}$ due to the emission of phonons, we have considered the theory developed by Brandes et al., [12, 13]. Including piezoelectric and deformation potential acoustic phonons, the transition rate reads:

$$W_{1,2}^{ph} = \frac{\pi T_{12}^2}{\hbar}\left[\frac{\alpha_{pie}}{\varepsilon} + \frac{\varepsilon}{\hbar^2 w_\xi^2}\right]\left[1 - \frac{w_d}{w}\sin\frac{w}{w_d}\right] \tag{4}$$

where α_{pie} is a piezoelectric coupling parameter, $\varepsilon = \hbar w = \mu_1 - \mu_2$, $w_d = c/d$ being c the sound velocity and d the distance between the dots. Finally,

$$\frac{1}{w_\xi^2} = \frac{1}{\pi^2 c^3}\frac{\Xi^2}{2\rho_M c^2 \hbar} \tag{5}$$

where ρ_M is the mass density and Ξ is the deformation potential. In Fig. 1, we represent schematically the inelastic contribution to I through the emission of phonons, between the corresponding levels of each QD.

Finally, we calculate the electronic spin-flip scattering rate $W_{i,j}^{sf}$ using a microscopic model that accounts for HF interactions and external magnetic fields:

$$\hat{H} = g_e \mu_B \vec{S}\cdot\vec{B} + \frac{A}{N_{L(R)}}\sum_{i=1}^{N_{L(R)}}\left[S_z I_z^i + \frac{1}{2}(S_+ I_-^i + S_- I_+^i)\right] \tag{6}$$

where A is the average HF coupling constant and I the nuclear spin . $N_{L(R)}$ is the number of nuclei in the left (right) dot. For simplicity we assume that $I = 1/2$. We take B to be oriented along the $\hat{z}$ direction (current direction). The HF interaction can then be separated into mean-field and flip-flop contributions:

$$\hat{H} = \hat{H}_z + \hat{H}_{sf} \tag{7}$$

where

$$\hat{H}_z = [g_e \mu_B B + A\langle I_z\rangle_{L(R)}]S_z \tag{8}$$

being,

$$\begin{aligned}\langle I_z\rangle_{L(R)} &= \frac{1}{N_{L(R)}}\sum_{i=1}^{N_{L(R)}}(I_z^i)_{L(R)} \\ &= \left[\frac{N^\uparrow - N^\downarrow}{N^\uparrow + N^\downarrow}\right]_{L(R)}|I_z| \\ &= P_{L(R)}|I_z| \end{aligned} \tag{9}$$

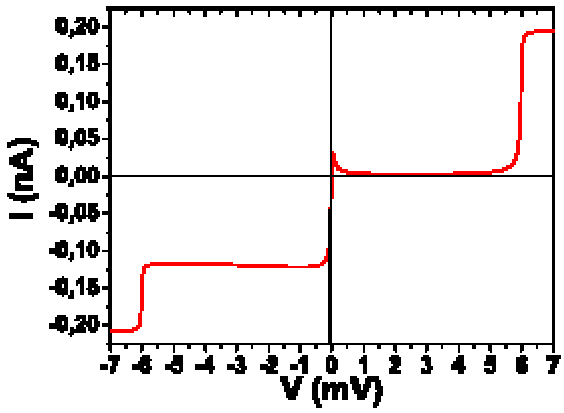

Figure 1: Stationary I/V_{DC} (B=0). At low V_{DC}, I flows as $|\downarrow,\uparrow\rangle$ is in resonance with $|0,\downarrow,\uparrow\rangle$. Once one electron tunnels from the emitter contact to the left dot with the same spin polarization as the electron in the right dot, the current drops abruptly due to spin blockade. In this region of bias voltage, the electronic charge is practically shared between $|\uparrow,\uparrow\rangle$ and $|\downarrow,\downarrow\rangle$ states (small finite occupation of other states, in particular of $|0,\downarrow,\uparrow\rangle$ occurs due to spin-flip induced by hyperfine interaction, giving rise to a finite current). At large V_{DC} the chemical potential of the right lead crosses the inter-dot triplet state and the right QD becomes suddenly discharged producing a large peak in the current. Sweeping backwards the DC voltage to negative values, electrons with spin up or down can flow from the right to the left quantum dot and spin blockade does not occur.

$P_{L(R)} = \left[\frac{N^{\uparrow}-N^{\downarrow}}{N^{\uparrow}+N^{\downarrow}}\right]_{L(R)}$ is the nuclear spin polarization where $N^{\uparrow(\downarrow)}$ is the number of nuclei with spin up(down), in a QD. We have chosen that initially, the nuclei polarization of the left and right dots are equal to zero.

$\hat{H}_z$ has external and effective nuclear field contributions. The latter given by:

$$B_{nuc} = \frac{A\langle I_z\rangle_{L(R)}}{g_e\mu_B} \tag{10}$$

On the other hand:

$$\hat{H}_{sf} = \frac{A}{2N_{L(R)}}\sum_i \left[S_+ I_-^i + S_- I_+^i\right] \tag{11}$$

is the flip-flop interaction responsible for mutual electronic and nuclear spin flips. Nuclei in similar quantum dots can give rise to different effective nuclear fields. This can be related with different effective hyperfine interactions in each dot. An slightly distinct number of nuclei can explain the different hyperfine interactions and eventually the independent behavior in terms of the effective nuclear fields. This is the case that we have considered : N_R ten per cent larger than N_L.

As discussed by Vagner and coworkers for 2DEG's, because of the mismatch between nuclear and electronic Zeeman energies spin-flip transitions must be accompanied at low temperature by phonon emission [6-11,14]. Phonon absorption rate is negligible at low temperatures and it can be neglected. Therefore for $B \neq 0$, hyperfine interaction only produces electronic spin-flip *relaxation* processes. We approximate the current-limiting spin-flip transition rate from parallel-spin to opposite-spin configurations by:

$$\left[\frac{1}{\tau_{sf}}\right]_{L(R)} \simeq \frac{2\pi}{\hbar} |<\hat{H}_{sf}>|^2 \frac{\gamma}{(\epsilon_i - \epsilon_f)^2 + \gamma^2} = \frac{2\pi}{\hbar} |<\hat{H}_{sf}>|^2 \frac{\gamma}{(\Delta Z_e)^2_{L(R)} + \gamma^2} \tag{12}$$

where the width γ is the electronic state life-time broadening which is of the order of μeV, i.e., of the order of the phonon scattering rate [2]. This equation shows that a different number of nuclei or different splitting Zeeman can give rise to a different spin-flip rate in each dot. The splitting Zeeman is given by:

$$(\Delta Z_e)_{L(R)} = g_e \mu_B B + \frac{A}{2} P_{L(R)} \tag{13}$$

is the total electronic Zeeman splitting including the Overhauser shift produced by the effective nuclear B.

$$(\Delta Z_{Overhauser})_{L(R)} = \frac{A}{2} P_{L(R)} \tag{14}$$

We assume weakly coupled QD's and we neglect exchange interaction. Then, within this approximation, the energy difference between initial and final states is given by the total Zeeman splitting. The time evolution of the nuclei spin polarization for both dots include the flip-flop interaction and a phenomenological nuclear spin relaxation time $\tau_{relax} \approx 100s$ [15] for the scattering between nuclei:

$$\dot{P}_L = W^{sf}_{6,3}\rho_3 - W^{sf}_{5,4}\rho_4 - \frac{P_L}{\tau_{relax}} \tag{15}$$

$$\dot{P}_R = W_{5,3}\rho_3 - W^{sf}_{6,4}\rho_4 - \frac{P_R}{\tau_{relax}} \tag{16}$$

where the expressions we propose for the the electronic spin-flip scattering rate $W^{sf}_{i,j}$ are given in [16].

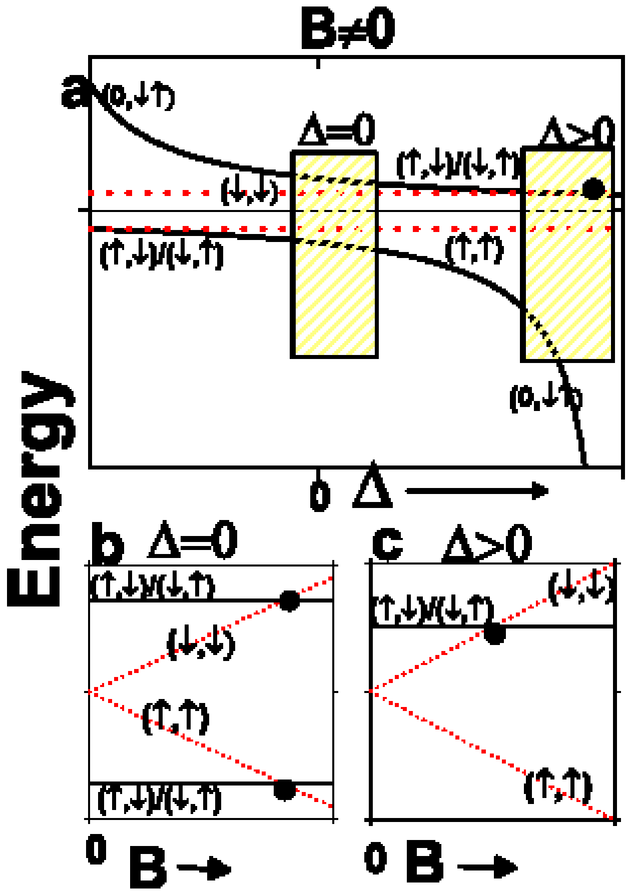

Figure 2: a): Energy levels diagram as a function of the detuning at finite B. At large detuning $|\downarrow,\downarrow\rangle$ is close to the $|\uparrow,\downarrow\rangle$ but electron-nuclei spin scattering is not efficient because it would imply phonon absorption which has very low probability at low temperature. Increasing B, both states cross. As $|\downarrow,\downarrow\rangle$ crosses the state $|\uparrow,\downarrow\rangle$, spin flip flop between electron and nuclei occurs and the current begins to flow. b) and c): Schematic diagrams showing the energy levels split versus B for zero and large detuning respectively.

Once we have obtained the time evolution for the electronic charge occupations and for the nuclei spin polarization of each quantum dot, we solve self-consistently the system of time differential equations and from the electronic charge occupations we calculate the tunneling current [16].

3 Results

We have calculated initially the tunneling current at zero external magnetic field as a function of the DC voltage. One can observe a resonance at low bias voltages which occurs due to the electron tunneling between $|\uparrow,\downarrow\rangle$ or $|\downarrow,\uparrow\rangle$ and $|0,\uparrow\downarrow\rangle$. Then, at certain time, states with electrons with same spin polarization ($|\uparrow,\uparrow\rangle$ and $|\downarrow,\downarrow\rangle$) become occupied and SB occurs. Increasing the forward bias voltage, there is a SB plateau region, where there is a slight current leakage due to finite spin-flip rate. At higher voltages, the right quantum dot becomes empty and the current flows increases abruptly. Sweeping backwards the voltage to negative values, we observe that there is not SB because the current occurs from the intradot double occupied singlet state of the right quantum dot $|0,\uparrow\downarrow\rangle$ to $|\uparrow,\downarrow\rangle$ or $|\downarrow,\uparrow\rangle$ and then the current is not suppressed. Similar results are showed in experiments [1].

Next, we have calculated the current at fixed bias voltage sweeping the external magnetic field. We have considered different density of nuclei within each quantum dot which implies that the HF interaction within each quantum dot is different. In Figure 2 we represent schematically the level diagram as a function of the detuning Δ between the electronic levels of the two quantum dots ϵ_L and ϵ_R. As Fig. 2 shows, depending of the detuning between the levels, the states $|\uparrow,\uparrow\rangle$, $|\downarrow,\downarrow\rangle$ are equidistant in energy (zero detuning), at a fixed B, with $|\uparrow,\downarrow\rangle$ or $|\downarrow,\uparrow\rangle$, states which, due to interdot tunneling, hybridize with the intradot singlet state $|0,\uparrow\downarrow\rangle$, and whose energy depends mainly on the interdot tunneling and the detunning Δ. For large detunings, $|\downarrow,\downarrow\rangle$ is closer in energy to the singlet state than $|\uparrow,\uparrow\rangle$. We have considered this particular case: increasing B, $|\downarrow,\downarrow\rangle$ becomes closer in energy to the singlet state but spin flip is not efficient up to the crossing with the singlet state. Once $|\downarrow,\downarrow\rangle$ has larger energy than the singlet, the process of spin-flip is energetically favorable and current flows (see Fig. 3). At the same time, dynamical nuclei polarization is induced for each quantum dot (see Fig. 4).

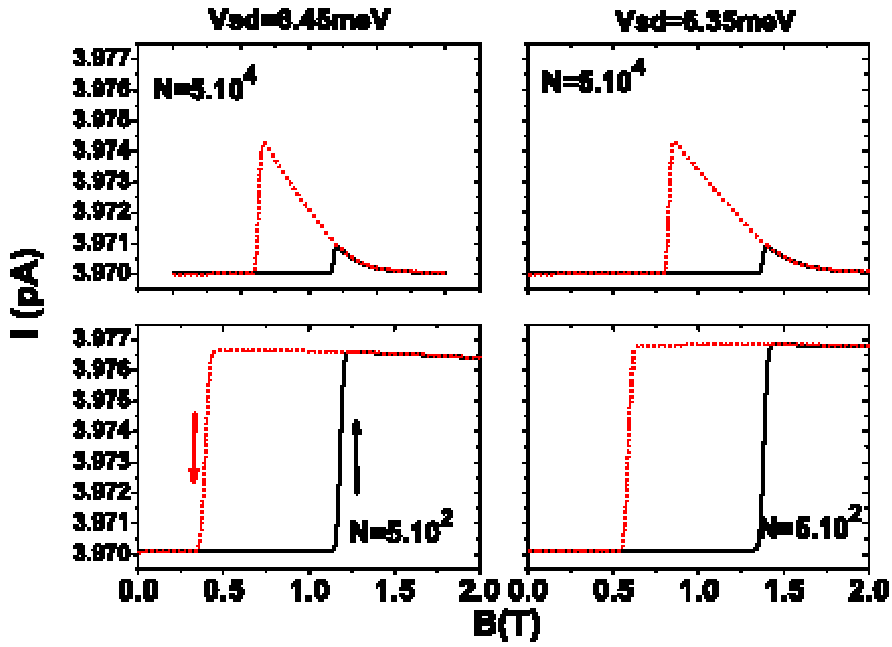

Figure 3: Stationary I/B curve at fixed V_{DC} for different densities of nuclei in the right quantum dot (the density of nuclei in the left dot remains always ten per cent smaller than in the right one). We observe a different current shape for different nuclei densities (different HF interaction intensities). In both cases, current is bistable due to the feedback between the electronic charge and the nuclei spin polarization. At lower densities of nuclei (bottom panels) spins current hysteresis presents a rectangular like shape whereas at higher densities (top panels) it presents a triangular like shape. According to our model [16] the HF interaction is inversely proportional to number of nuclei (see Eqs 11 and 12). Therefore a higher number of nuclei gives a weaker leakage current in terms of step (triangular versus rectangular) and hysteretic behavior (smaller hysteretic region). Experimental evidence[18] indicates a rectangular current step, as in the case presented here with a smaller number of nuclei in each dot.

Sweeping back the magnetic field, the calculated current and nuclei spin polarization present hysteretic behavior because of the strong feed-back between the electronic charge occupation which dynamically changes in the tunneling process and the dynamical nuclei polarization by the HF interaction induced in each quantum dot. It is similar to the effect of electronic charge flowing through a resonant nanostructure, as for instance a double barrier, which induces, by Coulomb interaction, an additional electrostatic field which itself modifies the electrostatic confining potential felt by the electron and then the tunneling current which presents bistability [17]

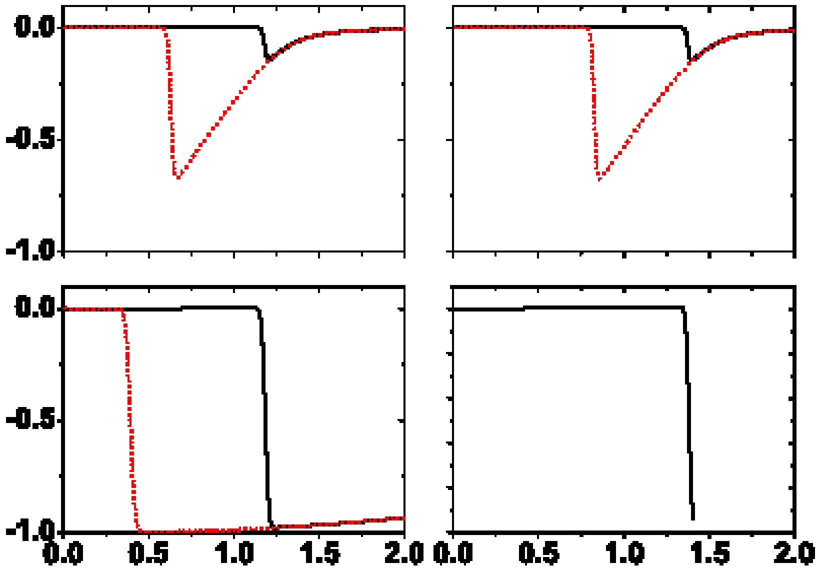

Figure 4: Nuclear polarizations of the right quantum dot versus applied B at two different fixed DC bias voltage and for two different densities of nuclei in the right QD (density in the left QD remains ten per cent smaller than in the right QD), i.e., for two different hyperfine interaction intensities. The behavior of P),at fixed bias voltage, as B changes is opposite to the electronic current behavior: sweeping forwards B the nuclei polarization drops below to zero as the triplet state $|\downarrow,\downarrow\rangle$ crosses the singlet state. Then, due to HF interaction, one electron flips its spin from down to up and the nuclei spin flips from up to down, giving rise to negative nuclei spin polarization. As the magnetic field is swept backwards, the energy crossing of levels occurs at different magnetic field intensity as sweeping forwards. Then, P presents hysteresis. This behavior occurs for both cases with different nuclei densities, however, at higher density P presents a triangular shape whereas al lower density it presents a rectangular shape.

In conclusion we calculate the magneto-tunneling current through a double quantum dot in the spin blockade regime and in presence of hyperfine interaction. We obtain strong non linear effects in the current as a result of the strong feedback between the electronic charge occupations and the spin polarization of the nuclei within each quantum dot. We show how changing the density of impurities one order of magnitude, the current shape is different for the two cases analyzed but, in both cases it presents bistability. A similar behavior has been extracted from our calculations for nuclei polarization. It shows that dynamical nuclei polarization plays an important role in spin blockade in quantum dots.

This paper is devoted to the memory of Israel Vagner. Israel was a very good and enthusiastic physicist and part of his work was devoted to the effect of magnetic fields in semiconductors and in particular to the analysis of nuclear spin relaxation. His work is a referent one in a topic which, nowadays a large activity is devoted to. Israel was not only interested in physics. He had other abilities as for instance the poetry.

This work has been supported by the MCYT (Spain) under grant MAT-2005-06444 (JI and GP), by the Ramón y Cajal program (J.I.) by the EU Human Potential Programme: HPRN-CT-2000-00144.

References

[1] K. Ono, D.G. Austing, Y. Tokura, and S. Tarucha, Science **297**, 1313 (2002).

[2] T. Fujisawa, D.G. Austing, Y. Tokura, Y. Hirayama, and S. Tarucha, Nature (London) **419**, 278 (2002); R. Hanson, B. Witkamp, L.M.K. Vandersypen, L.H. Willems van Beveren, J.M. Elzerman, and L.P. Kouwenhoven, Phys. Rev. Lett. **91**, 196802 (2003); J.M. Elzerman, R. Hanson, L.H. Willems van Beveren, B. Witkamp, L.M.K. Vandersypen, and L.P. Kouwenhoven, Nature **430**, 431 (2004).

[3] O. Gywat, H-A. Engel, D. Loss, R.J. Epstein, F.M. Mendoza, and D.D. Awschalom, Phys. Rev. B **69**, 205303 (2004).

[4] V. N. Golovach, A. Khaetskii, and D. Loss, Phys. Rev. Lett. **93**, 016601 (2004).

[5] S.I. Erlingsson, Y.V. Nazarov, and V.I. Fal'ko, Phys. Rev. B **64**, 195306 (2001); S.I. Erlingsson and Y.V. Nazarov, Phys. Rev. B **66**, 155327 (2002); S. I. Erlingsson, O.N. Jouravlev, and Y.V. Nazarov, Phys. Rev. B **72**, 033301 (2005); A.V. Khaetskii and Y.V. Nazarov, Phys. Rev. B **61**, 12639 (2000).

[6] S.V. Iordanskii, S.V. Meshkov, and I.D. Vagner, Phys. Rev. B **44**, 6554 (1991).

[7] I.D. Vagner and T. Maniv, Phys. Rev. Lett. **61**, 1400 (1988).

[8] D. Mozyrsky, V. Privman, and I.D. Vagner, Phys. Rev. B **68**, 085313 (2001).

[9] P. Dahan and I.D. Vagner, Phys. Rev. B **72**, 115328 (2005).

[10] Y.V. Pershin, S.N. Shevchenko, I.D. Vagner, and P. Wyder, Phys. Rev. B **66**, 035303 (2002).

[11] J.H. Kim, I.D. Vagner, and L. Xing, Phys. Rev. B **49**, 16777 (1994).

[12] T. Brandes and B. Kramer, Phys. Rev. Lett. **83**, 3021, (1999); T. Brandes, Phys. Rep. **408**, 315 (2005).

[13] T. Brandes, R. Aguado, and G. Platero, Phys. Rev. B **69**, 205326 (2004).

[14] S.I. Erlingsson and Y.V. Nazarov, Phys. Rev. B **64**, 195306 (2001).

[15] K. Ono and S. Tarucha, Phys. Rev. Lett. **92**, 256803, (2004); F.H.L. Koppens *et al.* Science, **309**, 1346 (2005).

[16] J. Iñarrea, G. Platero, and A.H. MacDonald, Phys. Rev. B **76**, 085329 (2007); Physica Status Solidi (a) **203**, 1148, (2006).

[17] J. Iñarrea and G. Platero, Europhys. Lett. **33**, 477 (1996); G. Platero and R. Aguado, Phys. Rep. **395**, 1 (2004).

[18] K. Ono and S. Tarucha, Phys. Rev. Lett. **92**, 256803 (2004).