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Low energy excitation spectrum in a layered superconductor vortex with small number of impurities in the vortex core *

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Abstract

The low energy exitation spectrum is found for a layered superconductor with small number of impurities inside the vortex core. All levels are found to be correlated. The level shift increases as the impurity comes closer to the vortex center. At some distance from the impurity to the vortex center levels practically cross. It is very important that all levels with small energy cross simultaneously. In such a case a statistical description of level position is impossible. If we neglect the weak repulsion of levels in this region, the positions of levels as a function of the distance from the vortex core to the impurity form two families of crossing straight lines. This leads to a strong enhancement of the conductivity in superclean layered superconductors.

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1 Introduction

The I-V characteristics of superconductors with weak pinning displays anomalous properties [1, 2]. Some of them are very difficult to explain in the framework of the quasiclassical approach. In quasiclassical approach there are three limiting cases, determined by the values of the parameters: the size of the gap Δ in the single-particle excitation spectrum, the level spacing ω_0 inside the vortex core $\omega_0 \sim \Delta^2/\varepsilon_F$ (ε_F - Fermi energy), and the electron mean collision time τ_{tr}. The three limiting cases are: 1) the dirty limit $\tau_{tr}\Delta \ll 1$, 2) the clean limit $\Delta \gg \tau_{tr}^{-1} \gg \omega_0$, and 3) superclean limit when the condition $\omega_0 \tau_{tr} \gg 1$ is fulfilled. In the dirty limit the vortex core is in a "normal" state in accordance with the picture of Bardeen and Stephen [3]. Bardeen and Sherman [4] and Larkin and Ovchinnikov [5] derived an expression for conductivity in a mixed state for low temperatures and small magnetic fields $B \ll H_{C2}$ in the case of moderately clean superconductors. In this case, compared to the previous picture, a logarithmically large factor arises in conductivity. This factor is related to shrinkage of vortex core at low temperatures $T \ll T_c$ [6].

The superclean case was studied in Ref. [7]. It was found that the level spacing ω_0 inside the vortex core plays the same role as the cyclotron frequency $\omega_c = eH/mc$ in a normal metal. It was also found, that in the superclean limit the Hall component of the conductivity tensor is the largest one $\sigma \approx en_e/B$, where n_e is the electron density in the conduction band and B is the magnetic field. The dissipative part of the conductivity tensor is smaller by a factor $(\omega_0 \tau_{tr})^{-1}$. Hence the dissipative part of the resistance tensor is the same as in moderately clean superconductors.

The quasiclassical approach is probably violated in the two-dimensional case (in layered superconductors), because the excitation spectrum in the vortex core is then discrete. Guinea and Pogorelov [8] considered the dissipation in the vortex state as a result of transitions between unperturbed levels, induced by "moving" impurities. Such a perturbation theory approach is valid only in the high velocity limit $V \gg v_F(\Delta/\varepsilon_F)^2$.

Here we consider the superclean limit. We have found, that in this region a new mechanism of dissipation arises [9]. In the superclean limit no more than one impurity can be found at distances of the order of the correlation length $\xi = v_F/\Delta$ from the vortex center (zero of Δ). In such a case, a statistical description of level position is not applicable. If an impurity is placed at a distance of order of ξ from the vortex center and is weak (Born parameter is small), then the level shifts are also small. It is important that levels with even and odd orbital momentums are shifted in opposite directions. The level shift increases as the impurity comes closer to the vortex center. At some distance from the impurity to the vortex center levels practically cross. It is very important that all levels with energy $|\varepsilon| \gg \Delta$ cross simultaneously.

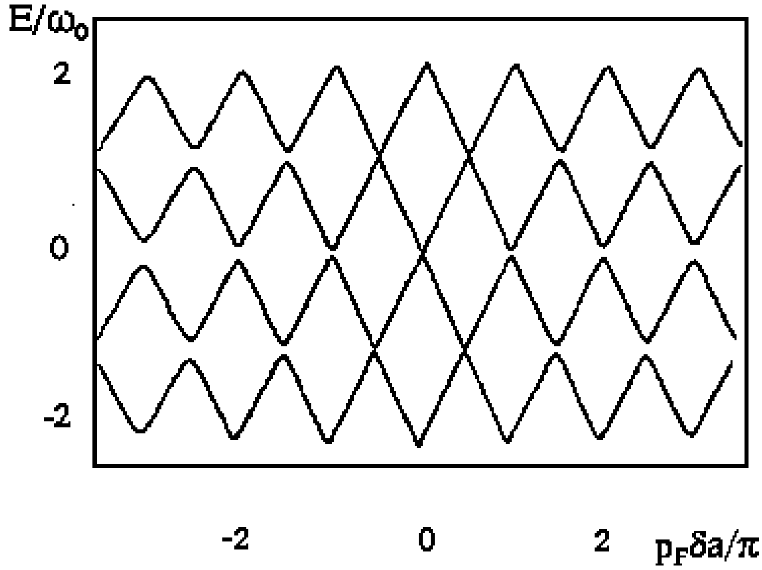

Figure 1: The excitation spectrum as a function of the impurity distance from the vortex center. The parameter $(\pi^2/2\omega_0 p_F)(\partial I_{1,am}/\partial a...)$ equals 0.02. The quantity $\delta a = a - a_0$, with a_0 given by Eq. (19).

If we neglect the weak (of order $\omega_0/p_F\xi$) repulsion of levels in this region, then position of levels as a function of the distance from the vortex core to the impurity form two families of crossing straight lines. Outside the dangerous level crossing region these lines are practically horizontal (see Fig. 1). The size of the dangerous zone, where the level lines can be considered as crossing, depends on the Landau-Zener parameter and hence on the vortex velocity. For the vortex velocity V in the range $\omega_0 \gg p_F V \gg \omega_0(\Delta/\varepsilon_F)^2$ the distribution function of excitations inside the core of the vortex does not change until the impurity comes into the dangerous zone. It changes essentially when the impurity goes through the dangerous zone. Excitations which arise when the impurity goes through this zone determine the value of the dissipative part of the conductivity. Such a mechanism of dissipation is essential for the electrical field E lying in the range

$$B\frac{v_F}{c}\left(\frac{\Delta}{\varepsilon_F}\right)^2 \gg E \gg B\frac{v_F}{c}\left(\Delta/\varepsilon_F\right)^3,$$

where c is the velocity of light. As it was found in [9], this mechanism of dissipation leads to the dissipative part of the current density that is given by

$$j_x = \frac{a_0 n_{imp}}{\phi_0}\,\frac{\varepsilon_F^{5/3}}{\Delta^{2/3}}\left(\frac{E}{v_F B}\right)^{2/3}, \tag{1}$$

where a_0 is the distance from the "dangerous" region to the vortex center $a_0 \sim \theta\xi$, and θ is the Born parameter which is equal to the phase shift of an electron scattering off the impurity. Usually $\theta \sim 1$. Hence $(p_F a) \gg 1$, and the current density essentially exceeds the value obtained in the framework of the quasiclassical approximation. In the range $p_F a \gg \omega_0 \tau_{tr} \gg 1$ the Hall angle is small.

2 The Low-Energy excitation spectrum for an impurity at the distance a from the vortex center in the range $a \gg \xi(\Delta/\varepsilon_F)^{1/2}$

The excitation spectrum E in the vortex state can be found as a solution of the eigenvalue problem for the following system of equations [10, 11]

$$\begin{pmatrix} -\frac{1}{2m}\frac{\partial^2}{\partial \mathbf{r}^2} - \mu + V(\mathbf{r}) - E; & \Delta(\mathbf{r}) \\ \Delta^*(\mathbf{r}); & \frac{1}{2m}\frac{\partial^2}{\partial \mathbf{r}^2} + \mu - V(\mathbf{r}) - E \end{pmatrix}\begin{pmatrix} f_1 \\ f_2 \end{pmatrix} = 0, \tag{2}$$

where Δ is the order parameter, μ is the chemical potential, and $V(\mathbf{r})$ - is the potential of impurities. We suppose here that the magnetic field B is weak $(B \ll H_{C2})$ and omit the vector potential in Eq. (2). Below, we consider the two-dimensional case. We suppose also that impurities are short range.

In our problem the order parameter Δ in the absence of impurities is given by the expression

$$\Delta(\mathbf{r}) = \Delta(r)\exp(i\varphi),$$

where φ is polar angle and $r = |\mathbf{r}|$.

The low energy excitation spectrum E_n^0 in the absence of the impurity was found in [10]

$$E_n^0 = -(n - 1/2)\omega_0, \tag{3}$$

where

$$\omega_0 = \int\limits_0^\infty \frac{dr\Delta(r)}{p_F r}\exp(-2K(r)) \Bigg/ \int\limits_0^\infty dr\exp(-2K(r)), \tag{4}$$

$$K(r) = \int\limits_0^r dr_1 \Delta(r_1)/\upsilon_F$$

If Kramer-Pesh effect takes place then with logarithmic accuracy we obtain from Eq. (4)

$$\omega_0 = \frac{\Delta^2}{\varepsilon_F}\ln\left(\frac{\Delta}{T}\right), \qquad \Delta = \Delta_{(\infty)}. \tag{5}$$

The eigenfunction corresponding to the eigenvalue of (3) is

$$\bar{f}_n = \begin{pmatrix} f_1 \\ f_2 \end{pmatrix}_n = \tilde{C}\exp(-K(r)) \begin{pmatrix} e^{in\varphi}J_n(p_F r) \\ -e^{i(n-1)\varphi}J_{n-1}(p_F r) \end{pmatrix}, \tag{6}$$

where $\tilde{C}$ is the normalization constant J_n - Bessel function, $n = 0, \pm 1, \pm 2$.

System (2) possesses a very important property: if E is an eigenvalue with eigenfunction (f_1, f_2), then $-E$ is also an eigenvalue and the corresponding eigenfunction is $(f_2^*, -f_1^*)$. This property holds in a magnetic field too.

For the excitation spectrum E in the presence of impurities inside the vortex core, we obtain from Eq. (2) the following system of equations

$$\det\Big((\hat{\varepsilon} - E) + \hat{A}\Big) = 0, \tag{7}$$

where the operator $\hat{A}$ is given by its matrix elements. In basis (6) we have

$$A_{kn} = \left\langle \bar{f}_k^+ \begin{pmatrix} V(\mathbf{r}-\mathbf{a}); & 0 \\ 0; & -V(\mathbf{r}-\mathbf{a}) \end{pmatrix} \bar{f}_n \right\rangle. \tag{8}$$

For a single impurity **a** is the position of the impurity relative to the vortex center, and

$$\hat{\varepsilon}_{kn} = \delta_{kn} E_n^0. \tag{9}$$

In Eq. (8) the values of r such that $r \gg p_F^{-1}$ are essential. We can use therefore an asymptotic expansion of Bessel functions to find matrix elements A_{kn}. A simple calculation gives

$$A_{kn} = e^{i(k-n)\varphi_a} \left\{ I_1(a) \cos\left(\frac{\pi(n+k)}{2}\right) - I_2(a) \sin\left(\frac{\pi(n+k)}{2}\right) \right\}, \tag{10}$$

where φ_a is the polar angle of the vector **a** and the quantities $I_{1,2}$ are given by the equation

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \frac{C^2}{a}\, e^{-2K(a)} \int d^2 r V(\mathbf{r}) \begin{pmatrix} \sin\left(2p_F\left(a + \frac{(\mathbf{ar})}{a}\right)\right) \\ \cos\left(2p_F\left(a + \frac{(\mathbf{ar})}{a}\right)\right) \end{pmatrix}. \tag{11}$$

The normalization constant C is defined by:

$$C^2 = \left\{ 2\pi \int_0^\infty dr \exp(-2K(r)) \right\}^{-1}. \tag{12}$$

If there are several impurities inside the vortex core, then the operator $\hat{A}$ in Eq. (7) is the sum of $\hat{A}_i$ over all impurities. Therefore

$$A_{kn}^{\{a_i\}} = \sum_i A_{kn}(a_i), \tag{13}$$

where A_{kn} is given by Eq. (10).

It follows from Eqs. (10), (13) that the transition matrix elements A_{kn} are separable. Therefore A_{kn} can be expressed as a finite sum of terms of the type $\tilde{A}_k^j \tilde{B}_n^j$

$$A_{kn} = \sum_j \tilde{A}_k^j \tilde{B}_n^j. \tag{14}$$

As a result we can obtain an expression for the excitation spectrum in an explicit form. If only one impurity is placed inside the vortex core, then we obtain from Eqs. (7), (10), (14) the following equation for the excitation spectrum

$$\det\begin{pmatrix} 1+I_1\sum\limits_{L=-N}^{N+1}\frac{1}{\varepsilon_{2L}+E}; & I_2\sum\limits_{L=-N}^{N+1}\frac{1}{\varepsilon_{2L}+E} \\ I_2\sum\limits_{L=-N}^{N+1}\frac{1}{\varepsilon_{2L-1}+E}; & 1-I_1\sum\limits_{L=-N}^{N+1}\frac{1}{\varepsilon_{2L-1}+E} \end{pmatrix}. \tag{15}$$

For the linear spectrum given by Eq. (3), we obtain in the limit of $N\to\infty$

$$\sum_{L=-N}^{N+1}\frac{1}{\varepsilon_{2L}+E}=\frac{\pi}{2\omega_0}\cot\left(\pi\left(\frac{1}{4}+\frac{E}{2\omega_0}\right)\right), \tag{16}$$

$$\sum_{L=-N}^{N+1}\frac{1}{\varepsilon_{2L-1}+E}=-\frac{\pi}{2\omega_0}\cot\left(\pi\left(\frac{1}{4}-\frac{E}{2\omega_0}\right)\right).$$

Using Eq. (16), we reduce Eq. (15) to the form [9]

$$1+\frac{\pi^2((I_1)^2+(I_2)^2)}{4\omega_0^2}+\frac{\pi I_1}{\omega_0\cos(\pi E/\omega_0)}=0. \tag{17}$$

It follows from Eq. (17) that the low energy excitation spectrum is strongly correlated even in the presence of an impurity inside the vortex core. If E_0 is a spectrum point, then all solutions of Eq. (17) are given by the equation

$$E=\pm E_0+2\omega_0 N, \qquad N=0,\pm1,\pm2... \tag{18}$$

Hence, the discrete spectrum is given by two sets of equidistant points. Functions $I_{1,2}$ are periodic with period π/p_F, both defined by the same function with a shift by a quarter of the period.

The amplitude I_{1am} of these functions is a smooth function of the parameter a/ξ and is given by Eq. (11).

With an accuracy of $\omega_0(\Delta/\varepsilon_F)$, a point a_0 exists such that

$$I_{1.am}(a_0)=-\frac{2\omega_0}{\pi}; \qquad I_2(a_0)=0. \tag{19}$$

Hence, at points $a_0+\delta a$ given by equation

$$\delta a=\left(\frac{\pi}{p_F}\right)N; \qquad N=0,\pm1,\pm2,... \tag{20}$$

we have

$$E_0 = \frac{\delta a}{2}\left(\frac{\partial I_{1.am}}{\partial a}\right)_{a_0} . \tag{21}$$

Eq. (21) means that in the vicinity of the points of the trajectory of the vortex, given by Eq. (19), there is a set of points separated by the distance δa, where spectrum lines are practically crossing (see Fig. 1). We denote the area in the vicinity of such points as the dissipation region. If the impact parameter of the trajectory is smaller then some critical value, then on such a trajectory there are two dissipation regions. When the vortex moves through these two dissipation regions many excitations are created inside the vortex core. The contribution of these excitations to the dissipative part of conductivity is found in [9].

3 One impurity at small distances $a \ll \xi(\Delta/\varepsilon_F)^{1/2}$ from the vortex center

First of all, we shall consider one impurity with a short-range potential placed at the vortex center. At the distance $\rho \gg p_F^{-1}$ from the vortex center we can use, for the solution of Eq. (2), the quasiclassical approximation with the first order correction terms. Indeed this correction terms will give an expression for the spectrum. At small distances of order of p_F^{-1} we can omit the nondiagonal elements in Eq. (2). As a result we obtain the following expression for the spectrum

$$E_n = -(n-1/2)\omega_0 + \upsilon_F \tan\left(\frac{\theta_{n-1}-\theta_n}{2}\right) \Bigg/ \int\limits_0^\infty d\rho\ \exp(-2K(\rho)) , \tag{22}$$

where θ_n is the scattering phase in a state with angular momentum n in the presence of the impurity potential $V(r)$. The corresponding eigenfunction is given by the expression

$$\bar{f}_n = \begin{pmatrix} f_1 \\ f_2 \end{pmatrix} = \frac{\tilde{C}}{2} e^{-K(\rho)} . \tag{23}$$

$$\cdot \begin{pmatrix} e^{in\varphi}[J_n(p_F\rho+\theta_n) + J_n(p_F\rho+\theta_{n-1})] \\ -e^{i(n-1)\varphi}[J_{n-1}(p_F\rho+\theta_n) + J_{n-1}(p_F\rho+\theta_{n-1})] \end{pmatrix},$$

where $\tilde{C}$ is a normalization constant. Suppouse now that the impurity is placed in a distance $\mathbf{a}$ from the vortex center such that $|\mathbf{a}| \ll \xi(\Delta/\varepsilon_F)^{1/2}$. We make a transformation in Eq. (2) to the coordinate system with the origin at the impurity. Then we obtain

$$\left(\begin{array}{ll} -\frac{1}{2m}\frac{\partial^2}{\partial \mathbf{r}^2} - \mu + V(\mathbf{r}) - E; & |\Delta|e^{i\varphi} + \left(\mathbf{a}\frac{\partial}{\partial \mathbf{r}}\right)(|\Delta|e^{i\varphi}) \\ |\Delta|e^{-i\varphi} + \left(\mathbf{a}\frac{\partial}{\partial \mathbf{r}}\right)(|\Delta|e^{-i\varphi}); & \frac{1}{2m}\frac{\partial^2}{\partial \mathbf{r}^2} + \mu - V(\mathbf{r}) - E \end{array} \right) \left(\begin{array}{c} f_1 \\ f_2 \end{array} \right) = 0. \tag{24}$$

From Eq. (24) we obtain the following equation for the excitation spectrum

$$\det\left((\hat{\varepsilon} - E) + \hat{A}\right) = 0, \tag{25}$$

where $\hat{\varepsilon}_{kn} = E_n \delta_{nk}$ and E_n is given by Eq. (22). The operator $\hat{A}$ is given by the matrix elements $\hat{A}_{kn}$ in the basis defined in Eq. (23)

$$\hat{A}_{kn} = \left\langle \bar{f}_k^+ \left(\begin{array}{ll} 0; & \left(\mathbf{a}\frac{\partial}{\partial \mathbf{r}}\right)(|\Delta|e^{i\varphi}) \\ \left(\mathbf{a}\frac{\partial}{\partial \mathbf{r}}\right)(|\Delta|e^{-i\varphi}); & 0 \end{array} \right) \bar{f}_n \right\rangle . \tag{26}$$

A simple calculation with the help of Eqs. (23), (26) gives

$$\hat{A}_{kn} = -\pi a C^2 \int_0^\infty d\rho \frac{|\Delta(\rho)|}{\rho} e^{-2K(\rho)} \, . \tag{27}$$

$$\cdot \left\{ \delta_{k,n+1} e^{-i\varphi_a} \cos\left(\frac{\theta_{n-1} - \theta_{n+1}}{2}\right) + \delta_{k,n-1} e^{i\varphi_a} \cos\left(\frac{\theta_n - \theta_{n-2}}{2}\right) \right\},$$

where the constant C is given by Eq. (12). It follows from Eq. (27) that in the operator of Eq. (25) only the diagonal and near-diagonal elements are nonzero. To solve Eq. (25) we define the function $B(I, E, n)$ in the following manner

$$B(I, E, n-1) = -E_n - E - \frac{I^2 \cos^2\left((\theta_{n-1} - \theta_{n+1})/2\right)}{B(I, E, n)}, \tag{28}$$

where

$$I = \pi a C^2 \int\limits_0^\infty d\rho \, \frac{|\Delta|}{\rho} e^{-2K(\rho)}, \qquad I/\omega_0 = p_F a/2. \tag{29}$$

With the help of function B, the Eq. (25) for spectrum is reduced to the following simple form

$$\det \begin{pmatrix} B(I,E,1) & -Ie^{i\varphi_a}\cos(\frac{\theta_0-\theta_2}{2}) & 0 & 0 \\ -Ie^{-i\varphi_a}\cos(\frac{\theta_0-\theta_2}{2}) & E_0-E & -Ie^{i\varphi_a} & 0 \\ 0 & -Ie^{-i\varphi_a} & E_1-E & -Ie^{i\varphi_a}\cos(\frac{\theta_0-\theta_2}{2}) \\ 0 & 0 & Ie^{-i\varphi_a}\cos(\frac{\theta_0-\theta_2}{2}) & B(I,-E,1) \end{pmatrix} = 0 \tag{30}$$

Equation (28) means that the quantity $B(I,E,1)$ can be presented as an infinite fraction

$$B(I,E,1) = -E_2 - E - \cfrac{I^2\cos^2(\frac{\theta_1-\theta_3}{2})}{-E_3 - E - \cfrac{I^2\cos^2(\frac{\theta_2-\theta_4}{2})}{-E_4 - E - \cfrac{I^2\cos^2(\frac{\theta_3-\theta_5}{2})}{\ldots B(I,E,n+1)}}} \tag{31}$$

Fraction (31) converges very quickly, if for $B(I,E,n+1)$ the expression

$$B(I,E,n+1) = \left\{(n+1/2)\omega_0 - E + \sqrt{((n+1/2)\omega_0 - E)^2 - 4I^2}\right\}/2, \tag{32}$$

$$(n+1/2)\omega_0 \pm E \gg |I| \tag{33}$$

is used.

Suppouse now, that the impurity is of a small size, so that only S scattering is essential. Suppouse also that the impurity potential is of the order of the atomic one and hence the following inequality takes place

$$\varepsilon_0 = \Delta\tan\left(\frac{\theta_0}{2}\right) \gg \omega_0. \tag{34}$$

Then, in the first order approximation Eq. (30) for low-energy excitations decouples in two independent branches

$$B(I,E,1) = 0 \qquad \text{and} \qquad B(I,-E,1) = 0. \tag{35}$$

Hence we obtain two independent families of spectrum lines. In this approximation they will cross, and only in the next approximation with respect to the parameter $(\omega_0/\varepsilon_0)^2$ will a gap in the crossing points open.

For small values of E we have

$$B(I, E, 1) = B + \alpha E, \tag{36}$$

where

$$\alpha = \frac{\partial B(I, E, 1)}{\partial E}. \tag{37}$$

Substituting expression (36) into Eq. (30) we obtain the following equation for the lowest energy level near the crossing points

$$E^2 \alpha^2 \varepsilon_0^2 = B^2 [I \cos(\theta_0/2)]^2 + [\varepsilon_0 B + (I \cos(\theta_0/2))^2]^2. \tag{38}$$

Hence the value of the gap δ near the crossing point is equal to

$$\delta = \frac{|I \cos(\theta_0/2)|^3}{|\alpha \varepsilon_0| \sqrt{\varepsilon_0^2 + (I \cos(\theta_0/2))^2}}. \tag{39}$$

In Eq. (37) the quantity α should be taken at the point

$$\alpha = \left. \frac{\partial B(I, E, 1)}{\partial E} \right|_{B = -\frac{\varepsilon_0 [I \cos(\theta_0/2)]^2}{[I \cos(\theta_0/2)]^2 + \varepsilon_0^2}}. \tag{40}$$

The value of the gap δ is given (by the order of magnitude) by the equation

$$\delta \sim \Delta (a/\xi)^3. \tag{41}$$

Equation (30) for the spectrum can be reduced to the form

$$\left[\frac{I^2}{E_0} \cos^2 \left(\frac{\theta_0 - \theta_2}{2} \right) + \frac{B(I, E, 1) + B(I, -E, 1)}{2} \right]^2 - \tag{42}$$

$$- \left[\frac{B(I, E, 1) - B(I, -E, 1)}{2} - \frac{E I^2}{E_0^2} \cos^2 \left(\frac{\theta_0 - \theta_2}{2} \right)^2 \right] +$$

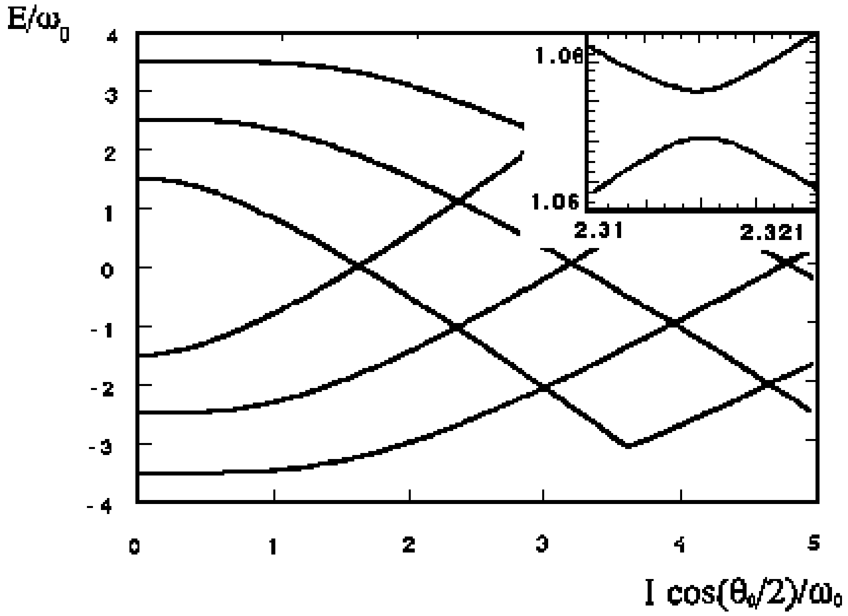

Figure 2: The low-energy excitation spectrum at small distances a from the impurity to the vortex center; $I/\omega_0 = p_F a/2$, see Eq. (28); $\theta \ll 1; E_0/\omega_0 = 50$.

$$+\frac{I^2-E^2}{E_0^2}B(I,E,1)B(I,-E,1)+\left[\frac{EI^2}{E_0^2}\cos^2\left(\frac{\theta_0-\theta_2}{2}\right)\right]^2=0.$$

Near the crossing points the last two terms are small (of order of $(I^3/E_0)^2$) and lead to the repulsion of spectrum lines.

Energy levels as a function of the shift $\mathbf{a}$ (or quantity $I \sim a$) are given in Fig. 2. In the inset of Fig. 2 the equation for two spectrum branches are

$$E/\omega_0 - 1.073 = \delta(0.1335t \pm \sqrt{1+t^2}), \tag{43}$$

$$t = 1.343(I/\omega_0 - 2.3172)/\delta,$$

$$\delta = 3.2 \cdot 10^{-3}.$$

4 Landau-Zener tunneling near the crossing point of spectrum lines

Near the crossing point of the spectrum lines, given by Eq. (16), we can put

$$\frac{\pi I_1}{2\omega_0} = -1 + y; \qquad \frac{\pi I_2}{2\omega_0} = 2p_F X; \qquad X = Vt, \tag{44}$$

where V is the velocity of a vortex. For two close spectrum points $E_\pm$ we obtain from Eq. (17) the following values

$$E_\pm = \pm\frac{\omega_0}{\pi}\varepsilon(t); \qquad \varepsilon(t) = \sqrt{y^2 + (2p_F X)^2}. \tag{45}$$

The usual Landau-Zener consideration leads to the following value W_{++} for a particle to remain on the same branch after collision [12]

$$W_{++} = 1 - \exp\left(-\frac{\omega_0 y^2}{2p_F|V|}\right). \tag{46}$$

Eq. (46) enables us to find the energy transferred to the vortex in each collision with an impurity.

5 Conclusion

In the two-dimensional case the excitation spectrum in vortex core is discrete. Then for small values of the vortex velocity V, such that

$$V \ll v_F(\Delta/\varepsilon_F)^2 \tag{47}$$

an excitation can arise only at spectrum line crossing points. Such crossing points are located only in dissipation regions, or if the vortex center is close to the impurity. For small impurities concentrations an impurity on the distances $a < \xi$ from the vortex center leads to the transition of quasiparticles to highly excited states. This leads to the strong enhancement of the conductivity in superclean layered superconductors [9].

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